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შინაარსი

The Stochastic Exponential as a Solution of Functional Equations

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1. Introduction. For a continuous semimartingale $X = (X_t, t \geq 0)$, with $X_0 = 0$, the stochastic exponential is defined as

$$\mathcal{E}_t(X) = e^{X_t - \frac{1}{2}\langle X \rangle_t}, \quad t \geq 0, \tag{1}$$

where $\langle X \rangle$ is the square characteristic of the martingale part of X.

$\mathcal{E}_t(X)$ is the unique solution of the linear stochastic differential equation

$$Z_t = 1 + \int_0^t Z_s dX_s, \quad t \geq 0$$

and in this sense $Z_t = \mathcal{E}_t(X)$ is the stochastic analogue of the usual exponential function $f(x) = e^{cx}$, which is the unique solution of the linear differential equation $f_x(x) = cf(x), f(0) = 1$.

On the other hand, it is well known that the property

$$e^{c(x+y)} = e^{cx}e^{cy}, \quad \text{for all } \ x, y \in R \tag{2}$$

is a characterizing property of exponential functions and in a very wide class of functions (e.g., in the class of measurable functions) $f(x) = e^{cx}$, where

$c \in R$ is some constant, is the general solution of the Cauchy exponential functional equation

$$f(x+y) = f(x)f(y) \quad \text{for all} \quad x, y \in R. \tag{3}$$

Similar to (2) property of the stochastic exponential (4) is the equality

$$\mathcal{E}_t(X)\mathcal{E}_t(Y) = \mathcal{E}_t(X + Y + \langle X, Y\rangle), \quad t \geq 0, \tag{4}$$

which is valid for any pair X, Y of semimartingales, where the additional term $\langle X, Y\rangle$ is the mutual characteristic of semimartingales X and Y. For the properties of stochastic exponential and for all unexplained notations from the martingale theory, we refer to [4] or [9] .

By definition, the stochastic exponential $\mathcal{E}_t(X)$ of a continuous semimartingale X can be expressed as a two-dimensional exponential function of a semimartingale and its square characteristic at time t

$$\mathcal{E}_t(X) = f(\langle X\rangle_t, X_t), \quad t \geq 0, \tag{5}$$

where $f(u, v) = e^{v-\frac{1}{2}u}, \quad u \geq 0, v \in R$. It follows from (4), that this function satisfies the following functional equation

$$f(\langle X\rangle_t, X_t)f(\langle Y\rangle_t, Y_t) = f(\langle X + Y\rangle_t, X_t + Y_t + \langle X, Y\rangle_t), \tag{6}$$

valid for any continuous semimartingales X and Y vanishing at 0. It is easy to verify that (6) is also true for the function

$$f(u, v) = e^{cv-\frac{c}{2}u}, \quad u \geq 0, v \in R, \tag{7}$$

for any constant $c \in R$.

Denote by $\mathcal{S}$ the class of continuous semimartingales and let $\mathcal{V}$ be a subclass of $\mathcal{S}$, for which equation (6) is satisfied. Following [7] we call the class $\mathcal{V}$ the domain of validity of equation (6).

One is led to the following question: given a class of two-dimensional functions (e.g., measurable, continuous), how small a class of semimartingales $\mathcal{V}$ can we take, so that the solutions of (6) remain to be only functions of the form (7).

We show in Theorem 1, that if equation (6) is satisfied for the class of stochastic integrals $h \cdot W$ with respect to the given Brownian Motion W and with deterministic integrands $0 \leq h \leq 1$, then the solution of equation (6)

is of the form (7). We prove this theorem by using the Cauchy exponential functional equation and the corresponding "almost" version.

If the domain of validity of equation (7) consists only of a single Brownian Motion W ($\mathcal{V} = \{W\}$), which corresponds (if we take $X = Y = W$ in (6)) to the equation

$$f^2(t, W_t) = f(4t, 2W_t + t), \quad t \geq 0, \tag{8}$$

then there exists a different from (7) continuous solution of equation (6) (see the counterexample after the proof of Theorem 1).

Note that the stochastic exponential $\mathcal{E}_t(X)$ transforms the class of continuous semimartingales into itself and this mapping is non-anticipative in the following sense: An adapted continuous process $F(t, X),\ t \geq 0$, depending on $X \in \mathcal{S}$, is non-anticipative, if for any continuous semimartingales X and Y and $t \geq 0$

$$F(t, X) = F(t, Y), \quad \text{when} \quad X_s = Y_s \quad \text{for all} \quad s \leq t.$$

Therefore, it seems natural to consider a functional equation for stochastic exponentials in terms of non-anticipative functionals

$$F(t, X)F(t, Y) = F(t, X + Y + \langle X, Y \rangle), \tag{9}$$

where by the property (4) the stochastic exponential $\mathcal{E}_t(X)$ satisfies equation (9), which represents more general form of equation (6). But (9) will be not a characterizing property of the stochastic exponential, since there exists a whole class of solutions of (9) which are not stochastic exponentials. E.g., if

$$F(t, X) = e^{\int_0^t \mathcal{K}(t,s) d(X_s - \frac{1}{2}\langle X \rangle_s)}, \quad X \in \mathcal{S}, \tag{10}$$

where $(k(t, s), s \geq 0, t \geq 0)$ is bounded, measurable deterministic function, then it is easy to see that $F(t, X)$ defined by (10) satisfies (9), but such processes are not always stochastic exponentials, see Theorem 2 and the remark at the end of the paper. In Theorem 2 we prove that under some restriction on the class of non-anticipative functionals, the general solution of (9) is of the form (10).

2. Functional equation for a function of a semimartingale and its square characteristic. Let $W = (W_t, t \geq 0)$ be a standard Brownian Motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Let $F = (\mathcal{F}_t, t \geq 0)$

be a filtration satisfying the usual conditions of right-continuity and completeness. Let $F^W = (\mathcal{F}_t^W, t \geq 0)$ be the filtration generated by the Brownian Motion W.

Let $\mathcal{S}$ (resp. $\mathcal{M}$) be the class of continuous semimartingales (martingales) vanishing at 0.

Let $\mathcal{M}^W(I)$ be a sub-class of stochastic integrals $h \cdot W$ with respect to the Brownian Motion W with integrands h, such that $h_u = I_{[s \leq u \leq t]}, 0 \leq s \leq t$.

Let $(f(u,v), u \geq 0, v \in R)$ be a function of two variables . We consider the functional equation

$$f(\langle X \rangle_t, X_t) f(\langle Y \rangle_t, Y_t) = f(\langle X + Y \rangle_t, X_t + Y_t + \langle X, Y \rangle_t), \tag{11}$$

for any $X, Y \in \mathcal{V}$ and P- a.e. for each $t \geq 0$, where $\mathcal{V}$ is some class of continuous semimartingales, the domain of validity of equation (11).

Theorem 1. Let $(f(u,v), u \geq 0, v \in R)$ be a function of two variables. Then the following assertions are equivalent:

a) The function f is a continuous strictly positive solution of the functional equation (11) with the domain of validity $\mathcal{V} = \mathcal{S}$.

b) The function f is a continuous strictly positive solution of the functional equation (11) with the domain of validity $\mathcal{V} = \mathcal{M}^W(I)$.

c) The function f is of the form

$$f(u,v) = e^{cv - \frac{c}{2}u} \quad \text{for some constant} \quad c \in R.$$

If we shall consider only measurable solutions, then the following two conditions will be equivalent:

b′) The function f is a measurable strictly positive solution of the functional equation (11) with the domain of validity $\mathcal{V} = \mathcal{M}^W(I)$.

c′) The function $f = (f(u,v), u \geq 0, v \in R)$ coincides with the function

$$e^{cv - \frac{c}{2}u} \quad \text{for some constant} \quad c \in R \tag{12}$$

almost everywhere with respect to the Lebesgue measure on $R_+ \times R$.

Proof. The implication *a*) $\rightarrow$ *b*) is evident. Let us prove the implication *b*) $\rightarrow$ *c*).

It follows from equation (11) that for any bounded deterministic functions h and g

$$f\Big(\int_0^t h_u^2 du, \int_0^t h_u dW_u\Big) f\Big(\int_0^t g_u^2 du, \int_0^t g_u dW_u\Big) = \tag{13}$$

$$= f\Big(\int_0^t (g_u + h_u)^2 du, \int_0^t (g_u + h_u) dW_u + \int_0^t g_u h_u du \Big)$$

P- a.e. for each $t \geq 0$.

For any fixed pair $s \leq t$ if we take $h_u = I_{(u<s)}$ and $g_u = I_{(s \leq u \leq t)}$, from (13) we obtain that P-a.s.

$$f(s, W_s) f(t - s, W_t - W_s) = f(t, W_t). \tag{14}$$

From (14) we have that

$$0 = E I_{(f(s,W_s) f(t-s, W_t - W_s) \neq f(t, W_t))} = \tag{15}$$

$$= \int_R \int_R I_{(f(s,x) f(t-s,y) \neq f(t, x+y))} \rho(s, x) \rho(t - s, y - x) dx dy,$$

where $\rho(s, x) = \frac{1}{\sqrt{2\pi s}} e^{-\frac{x^2}{2s}}$.

Therefore, for any $s > 0, t > 0, s \leq t$

$$f(s, x) f(t - s, y) = f(t, x + y) \quad \text{for all} \quad x, y \in R \tag{16}$$

almost surely with respect to the Lebesgue measure on R^2.

If $f(t, x)$ is continuous, then (16) is fulfilled for all $s > 0, t > 0, s \leq t, x \in R, y \in R$ and f satisfies the two-dimensional Cauchy exponential functional equation. The general continuous solution of this equation is of the form (see, e.g., [1], [6])

$$f(t, x) = \exp\{cx + bt\} \quad \text{for some} \quad b, c \in R. \tag{17}$$

On the other hand inserting $X = W$ and $Y = W$ in (11) we have

$$f^2(t, W_t) = f(4t, 2W_t + t) \quad t \geq 0 \tag{18}$$

P-a.s. and substituting $f(t, x) = \exp\{cx + bt\}$ in (18) we obtain that

$$\exp\{2cW_t + 2bt\} = \exp\{2cW_t + ct + 4bt\},$$

which implies that $b = c/2$, hence f is of the form $\exp\{cx - \frac{c}{2}t\}$.

$c) \to a)$ is evident. Indeed, if $f(u, v) = \exp\{cv - \frac{c}{2}u\}$ for some $c \in R$, then for any $X, Y \in \mathcal{S}$

$$f(\langle X \rangle_t, X_t) f(\langle Y \rangle_t, Y_t) = \exp\{cX_t - \frac{c}{2}\langle X \rangle_t\} \exp\{cY_t - \frac{c}{2}\langle Y \rangle_t\} =$$

$$= \exp\{c(X_t + Y_t) - \frac{c}{2}\langle X+Y\rangle_t + c\langle X, Y\rangle_t\} = f(\langle X+Y\rangle_t, X_t + Y_t + \langle X, Y\rangle_t)$$

P-a.s. for any $t \geq 0$.

The proof of the second part of Theorem 1 is similar if we use corresponding results on "almost" solutions of equation (16).

$b') \to c')$. It follows from results of [2] and [5] that (29) implies

$$f(t,x) = \exp\{cx + bt\} \quad \text{for some} \quad b, c \in R, \tag{19}$$

almost everywhere with respect to the Lebesgue measure on $R_+ \times R$.

From (18) we have that

$$f^2(t,x) = f(4t, 2x+t) \quad t \geq 0 \tag{20}$$

almost everywhere with respect to the Lebesgue measure on $R_+ \times R$. Therefore, from (19) and (20), avoiding three null sets, we obtain that

$$\exp\{2cx + 2bt\} = \exp\{2cx + ct + 4bt\},$$

almost everywhere with respect to the Lebesgue measure on $R_+ \times R$, which implies that $b = c/2$.

$c') \to b')$. Since for any $M \in \mathcal{M}^W(I)$ the random variable M_t admits Gaussian distribution, it follows from $c')$ that

$$f(\langle M\rangle_t, M_t) = \exp\{cM_t - \frac{c}{2}\langle M\rangle_t\} \tag{21}$$

P-a.s. for any $t \geq 0$, which will imply this implication, similarly to the proof of corresponding implication in first part of Theorem 1.

□

Remark. Note that, equality (21) can not be deduced from condition $c')$ for a general continuous semimartingale X. Hence, it does not follow from this condition that equation (11) is satisfied for all $X, Y \in \mathcal{S}$.

If the domain of validity of equation (11) consists only of a single Brownian Motion W ($\mathcal{V} = \{W\}$), which corresponds to the equation

$$f^2(t, W_t) = f(4t, 2W_t + t), \quad t \geq 0, P - a.s. \tag{22}$$

then there exists a different from (7) continuous solution of equation (22). Let $g(t,x) = \ln f(t,x)$. Then (22) is equivalent to equation

$$g(t, W_t) = \frac{1}{2} g(4t, 2W_t + t), \quad t \geq 0. \tag{23}$$

Let $g(t,x) = |(2x-t)t|^{\frac{1}{3}}$. It is evident, that this function satisfies equation (23), since

$$g(4t, 2x+t) = |(4x-2t)4t|^{\frac{1}{3}} = 2|(2x-t)t|^{\frac{1}{3}} = 2g(t,x).$$

Therefore, $f(t,x) = \exp|(2x-t)t|^{\frac{1}{3}}$ is a continuous solution of equation (22) different from (7).

3. An application to the Black-Scholes model. Now we give an example of application of Theorem 1 to the Black-Scholes model.

Let consider the Black-Scholes model, which consists of two securities. The first security, called a bond, with the price $(B_t, t \geq 0)$ defined by the differential equation

$$dB_t = rB_t dt, \;\; B_0 = b, \;\; r > 0, b > 0 \tag{24}$$

and the second security, a stock, whose price process satisfies the stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \quad S_0 = x > 0, \tag{25}$$

where $\mu, \sigma \in R, \sigma \neq 0$.

The solution of (24) is a continuously compound interest $B_t = be^{rt}$. If $F(t,x)$ is the amount to which a capital x increases during the time interval t by interest compounding, it was shown in [1] by using Cauchy's functional equations and some natural assumptions (axioms), that $F(t,x)$ should be equal to be^{rt}.

The unique solution of SDE (25) is represented as

$$S_t = ze^{\sigma W_t + (\mu - \frac{\sigma^2}{2})t}, \;\; S_0 = z. \tag{26}$$

We shall use similar arguments (as in [1]) to derive the representation (26) of the stock price process.

Let $f(t,z,w)$ be a measurable function of three variables and let $f(t,z,W_t)$ describes the price of a stock at time $t \geq 0$ with initial price z.

If we suppose that one can sell or buy any part of the stock, it will be natural to assume that $f(t,z,W_t)$ to which the initial price (capital) z evaluates during the time interval t, does not change by dividing the original initial capital $z = x + y$ into separate investments x, y. This assumption yields

$$f(t, x+y, W_t) = f(t,x,W_t) + f(t,y,W_t)$$

and according to the general measurable solution of the Cauchy additive functional equation

$$f(t,x,W_t) = C(t,W_t)x \tag{27}$$

It is also natural to assume that the capital that is evaluated during the time s from the initial capital x to $f(s,x,W_s)$ will amount to just as much after passing of the additional time t, as if the initial sum of money had been evaluated for the time $s+t$; thus by independent increment property of the Brownian Motion W and that $\tilde{W}_t = W_{t+s} - W_s, t \geq 0$ is a Brownian Motion starting from zero at the moment $t = 0$, we shall have that

$$f(s+t, x, W_{s+t}) = f(t, f(s,x,W_s), W_{t+s} - W_s), \tag{28}$$

which by (27) implies

$$xC(s+t, W_{s+t}) = f(s,x,W_s)C(t, W_{s+t} - W_s)$$

and, hence

$$C(s+t, W_{s+t}) = C(s,W_s)C(t, W_{s+t} - W_s),$$

which is the same as equality (14). It follows from here that for any $s > 0, t > 0, s \leq t$

$$C(s,x)C(t-s,y) = C(t, x+y) \quad \text{for all} \quad x, y \in R \tag{29}$$

almost surely with respect to the Lebesgue measure on R^2 (see the proof of Theorem 1).

Therefore, it follows from results of [2] and [5] that (29) implies

$$C(t,x) = \exp\{cx + bt\} \quad \text{for some} \quad b, c \in R, \tag{30}$$

almost everywhere with respect to the Lebesgue measure on $R_+ \times R$. Thus, for any $t \geq 0$

$$S_t = S_0 e^{cW_t + bt}, \quad P - \text{a.s.}$$

which is the same as (26) with $c = \sigma$ and $b = \mu - \sigma^2/2$.

Finally we notice that using the stochastic flow approach and some regularity assumptions (see [8]) it is also possible to derive equation (25) from the semigroup property (28).

4. A functional equation for non-anticipative functionals.

The mapping $h : [0,T] \times C[0,T] \to R$ is non-anticipative, if for any $\omega, \omega' \in C[0,T]$ and $t \in [0,T]$

$$h(t,\omega) = h(t,\omega'), \quad \text{when} \quad \omega_s = \omega'_s \quad \text{for all} \quad s \le t.$$

Consider the class of adapted continuous processes $F(t,X),\ t \ge 0$, which depend on $X \in \mathcal{S}$, defined by

$$\mathcal{C} = \{F;\ F(t,X) = e^{h(t,X-\frac{1}{2}\langle X\rangle)}, X \in \mathcal{S}, \text{for some continuous (in uniform norm),}$$
$$\text{non-anticipative functional } h(t,\omega)\}.$$

Theorem 2. The general solution of the functional equation

$$F(t,X)F(t,Y) = F(t,X+Y+\langle X,Y\rangle), \quad \text{for all} \quad X,Y \in \mathcal{S}, \tag{31}$$

in the class $\mathcal{C}$, is of the form

$$F(t,X) = e^{\int_0^t \mathcal{K}(t,s)d(X_s - \frac{1}{2}\langle X\rangle_s)}, \quad X \in \mathcal{S}, \tag{32}$$

where $(\mathcal{K}(t,s), s \ge 0, t \ge 0)$ is a deterministic function with $\mathcal{K}(t,s) = 0, t \le s$, such that

i) $\mathcal{K}(t,\cdot)$ is cadlag and has a finite variation, for each $t \in [0,T]$,

ii) $\int_0^T \omega_s \mathcal{K}(\cdot, ds)$ is continuous, for each $\omega \in C[0,T]$.

proof. Let F be a solution of equation (31) from the class $\mathcal{C}$. Then it follows from (31) and from the definition of the class $\mathcal{C}$ that

$$\begin{aligned} h(t,X) + h(t,Y) &= \ln F(t, X + \frac{1}{2}\langle X\rangle) + \ln F(t, Y + \frac{1}{2}\langle Y\rangle) \\ &= \ln F(t, X + Y + \frac{1}{2}(\langle X\rangle + \langle Y\rangle) + \langle X,Y\rangle) \\ &= \ln F(t, X + Y + \frac{1}{2}(\langle X+Y\rangle)) = h(t, X+Y) \end{aligned}$$

for any $X, Y \in \mathcal{S}$.

Since a deterministic function is a semimartingale if and only if it is of finite variation and the functions of finite variations are dense in C, it follows from the continuity of h that

$$h(t, \omega + \omega') = h(t, \omega) + h(t, \omega') \text{ for all } \omega, \omega' \in C. \tag{33}$$

By the Riesz theorem (see, e.g. [3]) for each t there exists a cadlag function $G(t, \cdot)$ of finite variation, such that $G(t, s) = G(t, T), s \geq t$ and

$$h(t, \omega) = \int_0^T \omega_s G(t, ds). \tag{34}$$

Using integration by part formula we get from (34) that

$$\begin{aligned} h(t, X) &= \int_0^t X_s G(t, ds) = X_t G(t, t) - \int_0^t G(t, s) dX_s \\ &= \int_0^t (G(t, t) - G(t, s)) dX_s = \int_0^t \mathcal{K}(t, s) dX_s, \end{aligned}$$

where $\mathcal{K}(s, t) = G(t, t) - G(t, s)$ is of finite variation for each t and by continuity of h the condition $ii)$ is also satisfied.

Now let us show that the non-anticipative functional

$$F(t, X) = e^{\int_0^t \mathcal{K}(t,s) d(X_s - \frac{1}{2}\langle X \rangle_s)}$$

satisfies equation (31). Let first show that F belongs to the class $\mathcal{C}$.

Consider the non-anticipative functional $h(t, \omega) = \int_0^T \omega_s \mathcal{K}(\cdot, ds)$, where $\mathcal{K}$ satisfies conditions $i) - ii)$ of the theorem. By $ii)$ this functional is continuous and the integration by part formula gives the equality

$$h(t, X) = -\int_0^t X_s \mathcal{K}(t, ds) = \int_0^t \mathcal{K}(t, s) dX_s, \ X \in \mathcal{S}. \tag{35}$$

Therefore, (35) implies that

$$F(t, X) = e^{\int_0^t \mathcal{K}(t,s) d(X_s - \frac{1}{2}\langle X \rangle_s)} = e^{h(t, X - \frac{1}{2}\langle X \rangle)}, \quad X \in \mathcal{S}, \tag{36}$$

which means that $F \in \mathcal{C}$.

It is evident that $F(t,X)$ defined by (32) satisfies (31), since for any continuous semimartingales X and Y

$$F(t,X)F(t,Y) = e^{\int_0^t k(t,s)d(X_s+Y_s-\frac{1}{2}\langle X\rangle_s-\frac{1}{2}\langle Y\rangle_s)} =$$

$$= e^{\int_0^t k(t,s)d(X_s+Y_s+\langle X,Y\rangle_s-\frac{1}{2}\langle X+Y\rangle_s)} = F(t,X+Y+\langle X,Y\rangle).$$

Remark . Note that $F(t,X)$ is not always a stochastic exponential. For example, if we take $\mathcal{K}(t,s) = v(t)$, where v is not of finite variation, then $F(t,X) = e^{v(t)(X_t-\frac{1}{2}\langle X\rangle_t)}$ will be not a stochastic exponential for any $X \in \mathcal{S}$. E.g., for $X = W$ the process $v(t)(W_t - \frac{1}{2}t)$ will be not a semimartingale and, hence $F(t,W)$ will not be a stochastic exponential.

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HADAMARD MATRICES AND SOME RELATED PROBLEMS

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Abstract. In this note, we consider a special case of the Hadamard matrix and prove one of its properties.

1. Introduction

There are various types of matrices in the literature having distinct properties useful for numerous applications, both practical and theoretical. The famous Hadamard matrix with orthogonality property was first defined by J.J. Sylvester [1] in 1867 and studied further by J.S. Hadamard [2] in 1893.

Definition 1.1. *A Hadamard matrix is a square matrix with entries equal to* ± 1 *whose rows (and hence columns) are mutually orthogonal.*

In other words, a Hadamard matrix of order n is a $\{1, -1\}$-matrix H_n satisfying the equality

$$H_n H_n^T = nI_n,$$

where I_n is the identity matrix of order n.

In 1867 Sylvester proposed a recurrent method for the construction of Hadamard matrices of order $2n$. Namely, if H_n is a Hadamard matrix of order n, then the matrix

$$\begin{bmatrix} H_n, & H_n \\ H_n, & -H_n \end{bmatrix}$$

is a Hadamard matrix of order $2n$.

There are several operations on Hadamard matrices which preserve the Hadamard property: (a) permuting rows, and changing the signs of some rows; (b) permuting columns, and changing the signs of some columns; (c) transposition.

Hadamard observed that a construction of Sylvester, mentioned above, produces examples of matrices that attain the bound when n is a power of 2, and produced examples of his own of sizes 12 and 20.

It is easy to prove that the order $n\,(n \geq 4)$ of any Hadamard matrix is divisible by 4. The converse is a long-standing conjecture.

Conjecture 1.1. *For every positive integer* n*, there exists a Hadamard matrix of order* $4n$*.*

Conjecture 1.1 is true for $4n = 2^k$. Currently unknown smallest order is $4n = 668$.

The problems considered here have been discussed by the authors more than once (see, for example, [3–5]).

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2. Sylvester matrices

The sequence of the matrices defined by the following recurrence relation:

$$S^{(0)} = [1], \quad S^{(n)} = \begin{bmatrix} S^{(n-1)}, & S^{(n-1)} \\ S^{(n-1)}, & -S^{(n-1)} \end{bmatrix}, \quad n = 1, 2, \dots$$

is a particular subclass of the class of Hadamard matrices and are named as the Sylvester (or Walsh) matrices. $S^{(n)} = \left[s_{ij}^{(n)}\right]$ is a square matrix of order 2^n, where $s_{ij}^{(n)} = \pm 1$. For example,

$$S^{(1)} = \begin{bmatrix} 1, & 1 \\ 1, & -1 \end{bmatrix}, \quad S^{(2)} = \begin{bmatrix} 1, & 1, & 1, & 1 \\ 1, & -1 & 1, & -1 \\ 1, & 1 & -1, & -1 \\ 1, & -1 & -1, & 1 \end{bmatrix}, \quad \text{and so on.}$$

Let $S^{(n)} = \left[s_{ij}^{(n)}\right]$ be a Sylvester matrix of order 2^n. Consider the following expression

$$\varrho^{(n)}(m) = \sum_{j=1}^{2^n} \left| \sum_{i=1}^{m} s_{ij}^{(n)} \right|, \quad m = 1, 2, \dots, 2^n, \tag{2.1}$$

and let

$$\varrho^{(n)} = \max_{1 \le m \le 2^n} \varrho^{(n)}(m).$$

It is easy to see that for any positive integer n we have $2^n \le \varrho^{(n)} \le 2^{2n}$.

For example, clearly

$$\varrho^{(2)}(1) = 4, \varrho^{(2)}(2) = 4, \varrho^{(2)}(3) = 6, \varrho^{(2)}(4) = 4.$$

Hence $\varrho^{(2)} = 6$.

To prove Theorem 2.2 we need the following elementary lemma.

Lemma 2.1. *The expression $\varrho^{(n)}(m)$, defined by equality (2.1), has the following properties:*

$$\varrho^{(n)}(2^{n-1} + m) = \varrho^{(n)}(2^{n-1} - m) + 2m \tag{2.2}$$

for any $1 \le m \le 2^{n-1}$, $n = 1, 2, \dots$ (here we assume that $\varrho^{(n)}(0) = 0$ for any n).

$$\varrho^{(n+1)}(m) = 2\varrho^{(n)}(m) \tag{2.3}$$

for any $1 \le m \le 2^n$, $n = 1, 2, \dots$.

$$\varrho^{(n)}(2^{n-1} + 2^{n-2} + m) = \varrho^{(n)}(2^{n-1} + 2^{n-2} - m) \tag{2.4}$$

for any $1 \le m \le 2^{n-2}$, $n = 2, 3, \dots$.

Our main goal is to prove the following theorem.

Theorem 2.2. *For every positive integer n we have*

$$\varrho^{(n)} = \max_{1 \le m \le 2^n} \varrho^{(n)}(m) = \frac{3n+7}{9} \cdot 2^n + (-1)^n \cdot \frac{2}{9}.$$

For any n the maximum is attained at the points

$$m_n = \frac{2^{n+1} + (-1)^n}{3} \quad \textit{and} \quad m_n' = \frac{5 \cdot 2^{n-1} + (-1)^{n-1}}{3}.$$

Proof. It is easy to see that $\frac{3n+7}{9}2^n + (-1)^n \frac{2}{9}$ is a positive integer for any positive integer n.

First let us prove by induction that

$$\varrho^{(n)} \le \frac{3n+7}{9}\, 2^n + (-1)^n \frac{2}{9}. \tag{2.5}$$

Indeed, for $n = 1$ and $n = 2$ the inequality (2.5) is evident.

Now suppose that $n \ge 3$ and let the inequality (2.5) be valid for some n. Let us prove its validity for $n+1$. Namely, we will show that

$$\varrho^{(n+1)} \le \frac{3n+10}{9}\, 2^{n+1} + (-1)^{n+1} \frac{2}{9}. \tag{2.6}$$

For this it suffices to show

$$\varrho^{(n+1)}(m) \le \frac{3n+10}{9}\, 2^{n+1} + (-1)^{n+1} \frac{4}{9} \quad \text{for any} \quad 1 \le m \le 2^{n+1}.$$

Let $1 \le m \le 2^n$.

Then using the property (2.3) and relation (2.5) we have

$$\varrho^{(n+1)}(m) = 2\varrho^{(n)}(m) \le \frac{3n+7}{9}\, 2^{n+1} + (-1)^n \frac{4}{9}. \tag{2.7}$$

It is easy to see that the right-hand side of (2.7) does not exceed the right-hand side of (2.6). Indeed, it is clear that the inequality

$$\frac{3n+7}{9}\, 2^{n+1} + (-1)^n \frac{4}{9} \le \frac{3n+10}{9}\, 2^{n+1} + (-1)^{n+1} \frac{2}{9}$$

is equivalent to the inequality $(-1)^n(2/3) \le (2/3)2^n$, which is true for all $n \ge 1$ and this case is proved.

Let $2^n + 1 \le m \le (5/4)2^n$.

Let us represent m in the following form $m = 2^n + \bar{m}$, where $1 \le \bar{m} \le 2^{n-2}$. Then by (2.2) and (2.3) we get:

$$\varrho^{(n+1)}(m) = \varrho^{(n+1)}(2^n + \bar{m}) = \varrho^{(n+1)}(2^n - \bar{m}) + 2\bar{m} = 2\varrho^{(n)}(2^n - \bar{m}) + 2\bar{m} \le$$

$$\le \frac{3n+7}{9}\, 2^{n+1} + (-1)^n \frac{4}{9} + 2^{n-1} = \frac{3n+9,25}{9}\, 2^{n+1} + (-1)^n \frac{4}{9}. \tag{2.8}$$

Now let us show that the right-hand side of (2.8) does not exceed the right-hand side of (2.6). Indeed, it is clear that the inequality

$$\frac{3n+9,25}{9}\, 2^{n+1} + (-1)^n \frac{4}{9} \le \frac{3n+10}{9}\, 2^{n+1} + (-1)^{n+1} \frac{2}{9}$$

is equivalent to the inequality $(-1)^n(2/3) \le (1/6)2^n$, which is true for all $n \ge 1$ and this case is proved.

Let $(5/4)2^n + 1 \le m \le (7/4)2^n$.

Let us represent m in the following form $m = 2^n + 2^{n-1} \pm \bar{m}$, where $1 \le \bar{m} \le 2^{n-2}$. Note that by (2.3) it is enough to consider the case $m = 2^n + 2^{n-1} - \bar{m}$. We have

$$\varrho^{(n+1)}(m) = \varrho^{(n+1)}(2^n + 2^{n-1} - \bar{m}) = 2\varrho^{(n)}(2^{n-1} + \bar{m}) + 2^n - 2\bar{m} =$$

$$= 2\left(\varrho^{(n)}(2^{n-1} - \bar{m}) + 2\bar{m}\right) + 2^n - 2\bar{m} = 4\left(\varrho^{(n-1)}(\bar{m}) + 2^{n-2} - \bar{m}\right) + 2^n + 2\bar{m} =$$

$$= 8\varrho^{(n-2)}(\bar{m}) + 2^{n+1} - 2\bar{m} \le \frac{3n+1}{9}\, 2^{n+1} + (-1)^{n-2}\frac{16}{9} + 2^{n+1} - 2. \tag{2.9}$$

Now let us show that right-hand side of (2.9) does not exceed the right-hand side of the inequality (2.6). Indeed, the inequality

$$\frac{3n+10}{9}\,2^{n+1}+(-1)^n\frac{16}{9}-2\le\frac{3n+10}{9}\,2^{n+1}+(-1)^{n+1}\frac{2}{9}$$

is equivalent to the inequality $(-1)^n 2\le 2$, which obviously is true for all $n\ge 1$ and, consequently, this case is proved.

Consider $(7/4)2^n+1\le m\le 2^{n+1}$.

Let us represent m in the following form $m=2^{n+1}-\bar{m}$, where $1\le\bar{m}\le 2^{n-2}$. We have

$$\varrho^{(n+1)}(m)=\varrho^{(n+1)}(2^{n+1}-\bar{m})=\varrho^{(n+1)}(\bar{m})+2^{n+1}-2\bar{m}=$$

$$=8\varrho^{(n-2)}(\bar{m})+2^{n+1}-2\bar{m}\le\frac{3n+1}{9}\,2^{n+1}+(-1)^{n-2}\frac{16}{9}+2^{n+1}-2. \qquad (2.10)$$

Let us show that the right-hand side of (2.10) does not exceed the right-hand side of the inequality (2.6). Indeed, clearly the inequality

$$\frac{3n+10}{9}\,2^{n+1}+(-1)^n\frac{16}{9}-2\le\frac{3n+10}{9}\,2^{n+1}+(-1)^{n+1}\frac{2}{9}$$

is equivalent to the inequality $(-1)^n 2\le 2$, which is true for any $n\ge 1$ and, therefore inequality (2.5) is completely proved.

Now let us consider the numbers $m_n=\frac{2^{n+1}+(-1)^n}{3}$ and $m_n^{'}=\frac{5\cdot 2^{n-1}+(-1)^{n-1}}{3}$. It is easy to verify that for any $n\ge 1$ both of these numbers are integers and $2^{n-1}\le m_n, m_n^{'}\le 2^n$.

Let us show by the induction the equality

$$\varrho^{(n)}(m_n)=\varrho^{(n)}\left(\frac{2^{n+1}+(-1)^n}{3}\right)=\frac{3n+7}{9}\,2^n+(-1)^n\,\frac{2}{9}. \qquad (2.11)$$

It is clear that $\varrho^{(1)}=2$ and $\varrho^{(1)}(m_1)=\varrho^{(1)}\left(\frac{4-1}{3}\right)=\varrho^{(1)}(1)=2$.

Now we assume that (2.11) is valid for some $n\ge 1$ and prove its validity for $n+1$. Thus we have to prove that

$$\varrho^{(n+1)}(m_{n+1})=\varrho^{(n+1)}\left(\frac{2^{n+2}+(-1)^{n+1}}{3}\right)=\frac{3n+10}{9}\,2^{n+1}+(-1)^{n+1}\,\frac{2}{9}.$$

Using (2.2) and (2.3) we have

$$\varrho^{(n+1)}(m_{n+1})=\varrho^{(n+1)}\left(2^n-\frac{2^n+(-1)^{n-1}}{3}\right)+2\cdot\frac{2^n+(-1)^{n-1}}{3}=$$

$$=2\varrho^{(n)}\left(\frac{2^{n+1}+(-1)^n}{3}\right)+2\cdot\frac{2^n+(-1)^{n-1}}{3}=\frac{3n+10}{9}\,2^{n+1}+(-1)^{n+1}\,\frac{2}{9}.$$

It remains to show that $\varrho^{(n)}(m_n)=\varrho^{(n)}(m_n^{'})$. Indeed, again according to (2.2) and (2.3) we have

$$\varrho^{(n)}(m_n^{'})=2\varrho^{(n-1)}\left(\frac{2^{n-1}-(-1)^{n-1}}{3}\right)+2\cdot\frac{2^n+(-1)^{n-1}}{3}=$$

$$=4\varrho^{(n-2)}\left(\frac{2^{n-1}+(-1)^{n-2}}{3}\right)+2\cdot\frac{2^n+(-1)^{n-1}}{3}=\varrho^{(n)}(m_n).$$

Thus Theorem 2.2 is completely proved. □

Let $S^{(n)} = \left[s_{ij}^{(n)}\right]$ be a Sylvester matrix of order 2^n and $(a_i)_{i\leq 2^n}$ be the sequence of vectors in 2^n-dimensional space $\mathbb{R}^{2^n}$ defined as follows

$$a_i = \left(s_{i1}^{(n)}, s_{i2}^{(n)}, \ldots, s_{i2^n}^{(n)}\right).$$

Let us formulate the assertion of Theorem 2.2 in the following manner

$$\varrho^{(n)} = \left\|\sum_{i=1}^{m_n} a_i\right\|_{l_1} = (3n+7)2^n/9 + 2(-1)^n/9,$$

where $m_n = (2^{n+1} + (-1)^n)/3$, and $\|\cdot\|_{l_1}$ is the l_1-norm in $\mathbb{R}^{2^n}$ (i.e. if $b = (\beta_1, \beta_2, \ldots, \beta_{2^n})$, then $\|b\|_{l_1} = \sum_{j=1}^{2^n} |\beta_j|$).

Let us consider a permutation $\pi : \{1, 2, \ldots, 2^n\} \to \{1, 2, \ldots, 2^n\}$ and the following expression:

$$\left\|\sum_{i=1}^{m_n} a_{\pi(i)}\right\|_{l_1}.$$

Denote by Π the set of all permutations of the sequence $\{1, 2, \ldots, 2^n\}$. By the property (a) of Hadamard matrices mentioned above the matrix $S_\pi^{(n)} = \left[s_{\pi(i)j}^{(n)}\right]$ is a Hadamard matrix as well.

Authors do not know yet the answer to the following conjecture:

Conjecture 2.1. *For any positive integer $n \geq 1$ and for any permutation $\pi \in \Pi$ we have*

$$\left\|\sum_{i=1}^{m_n} a_{\pi(i)}\right\|_{l_1} \geq (3n+7)2^n/9 + 2(-1)^n/9.$$

Note that we have conducted a lot of computer experiments. The results do not contradict Conjecture 2.1, though a theoretical proof is not known yet.

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Valuation of Hedging Strategies using Collars with Strike Prices Predicted by Machine Learning Models

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Introduction

This paper follows work on gold price risk management problem and valuation of investment in gold widgets production, where gold is an input asset hence introducing risk of increased gold prices. Previous work is discussed in [15] and further developed in [20] with introduction of Machine Learning (ML) models as tools for generating gold price predictions. As a result, strike price in hedging strategies is dictated by ML predictions. As discussed in [13] because marketed puts with arbitrary strike prices are not's guaranteed, a replicating portfolio principle is used to mimic payoff of the put on desired strike price. Based on gold price predictions, supposedly, optimal strike prices are selected, and, therefore, less costly protective put is constructed. This, in turn, reduces total cost of rebalancing replicating portfolio, and, thus, the strike price of real option in the investment project.

This paper attempts at improving previous works in [14, 15, 20] and increase value-add from hedging strategy by introducing collars instead of puts as a protection against gold price increase. To construct a collar, one needs both call and put option at different strike prices, generally, out-of-the-money strike prices are used and gap between them is directly tied to desired level of protection. Using an ensemble of ML models, this paper illustrates how lower and upper bounds of gold price predictions can be used as call and put strike prices, respectively. This approach, to the extent of predictions accuracy, ensures constructing cheapest collars within weekly rebalancing process of replicating portfolio that serves as a hedge against gold price increase. Hence hedging strategy using as cheap collars as possible is likely to maximize total value of investment in gold widgets production.

Review and summary of hedging strategies

This section follows practical example described in [12] with Georgian producer of gold widgets, Zarapkhana and its gold price risk hedging problem. Opposed to Robust Valuation techniques described in [1] and [12], where project valuation is required under incomplete markets scenario and it is focused on risk mitigation of non-tradable assets such as widgets produced by local manufacturers, following work in [14, 15, 20] focuses on hedging Gold price risk as an input cost for producing widgets. Below is summary of results obtained from different hedging strategies used in above mentioned previous works:

- **Protective put using at-the-money strike price**. Value of investing in the project using protective put hedging strategy with at-the-money strike price is

$$PV = DCF + Real\ Option\ Value = 44.74 + 42.42 = 87.16$$

- **Protective Put with variable strike price predicted by ML ensemble**. Using same example as above but varying the strike price given by ensemble of ML model predictions, investment project value is

$$PV = DCF + Real\ Option\ Value = 44.74 + 55.84 = 100.58$$

From above it is evident that variable strike price resulted into Real Option value add-in of 13.42 per gram. As varying strike prices were based on ML predictions, it begs a question whether more sophisticated hedging strategy like use of collars instead of only puts can bring even more Real Option Value to the investment project. To introduce this strategy an illustrative example of using collars to hedge against gold price risk at a single point in time is described in the next section.

Hedging strategy using short collar

As an illustration of hedging with a collar at any single point in time, this section follows example from Chapter 4 of reference [10], Introduction to Risk Management, specifically, the buyer's perspective. Auric Enterprises is a manufacturer of widgets, a product that uses gold as an input. Auric sells each widget for a fixed price of $800, fixed cost per widget is $340, and the manufacture of each widget requires 1 oz. of gold as an input, thus, unhedged profit is given by:

$$unhedged\ profit = P_{widget} - Cost - P_{gold} = \$800 - \$340 - P_{gold} = \$460 - P_{gold}$$

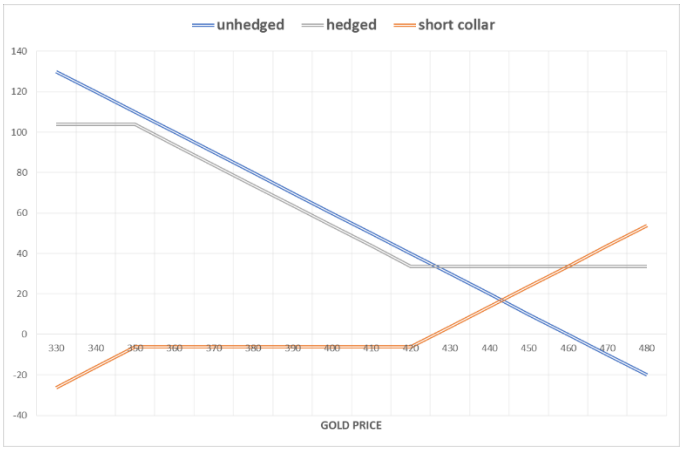
This implies a breakeven price of gold for Auric of $460, at any price above it, Auric will generate a loss. Now, let's assume Auric is not willing to pay more than $420 per oz. of gold and, also, believes that price of gold will not become less than $350. To fully leverage on its views, Auric can enter following options positions:

- Long call option on gold at strike price of $420
- Short put option on gold at strike price of $350

This, in turn, is short collar strategy. Thus, hedging with short collar using above strike prices and denoting future value of option premiums by *cp*, results into hedged profit, at any single point in time, equal to:

$$hedged\ profit = P_{widget} - Cost - P_{gold} + max(0, P_{gold} - \$420) - \max(0, \$350 - P_{gold}) - cp$$

Unhedged profit, hedge profit, and short collar itself are shown on the graph below.



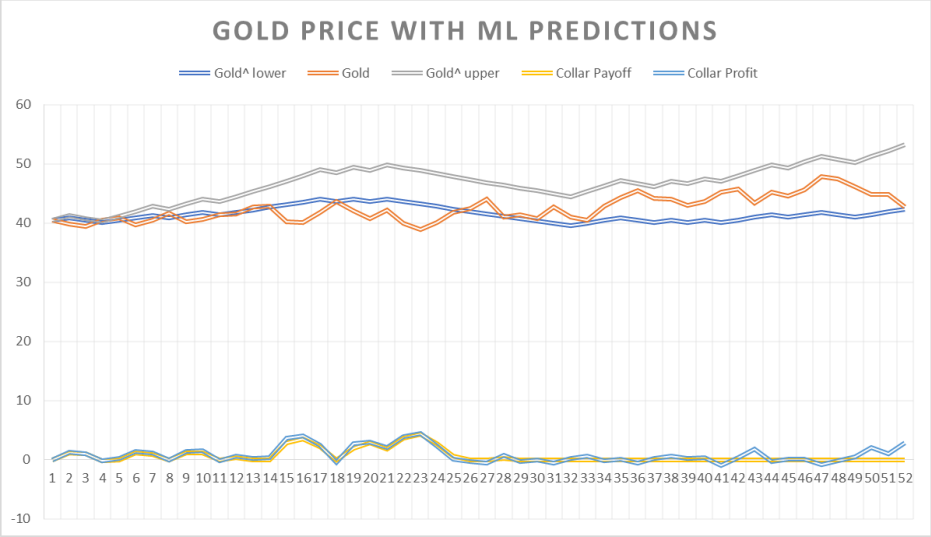
Following section describes choice of call and put option strike prices in short collar strategy that are based on lower and upper bounds of gold price predictions by an ensemble of ML models. Then collar constructed with these strikes is used in replicating portfolio as hedge against gold price movements.

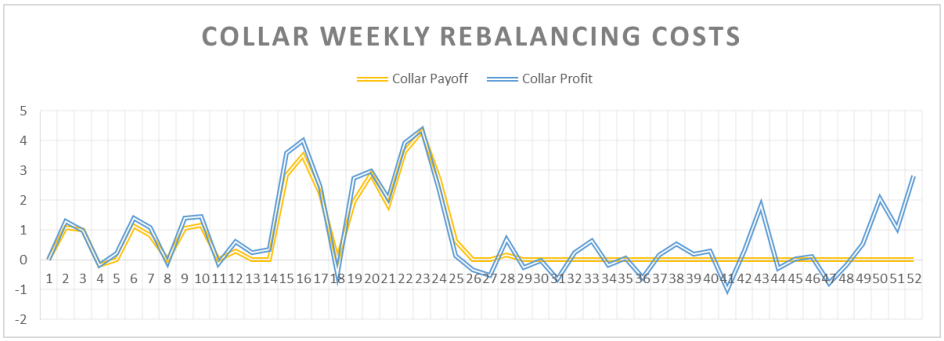
Incorporating ML predictions in hedging with short collar strategy

Previous section showed simplified example of hedging with short collar at a single point in time. This section continues practical example of Georgian producer of gold widgets, Zarapkhana, and describes short collar hedge with ML predictions. As mentioned above, marketed calls and puts with arbitrary strike prices are not likely to exist. Hence, weekly rebalanced replicating portfolio principle applied in [20] is used in conjunction with put-call parity to replicate both options that mimic payoff from short collar strategy. From previous section it is clear that cost of short collar strategy depends on the choice of strike prices, with larger gaps, i.e., more out-of-the-money options, costing less. Although cheaper protection insures smaller profits, an accurate view of future gold price movements can help to strike the balance between the two. In line with weekly rebalancing of short collar replicating portfolio, one can come up with weekly updated predictions of lowest and highest gold prices and use them as strike prices of calls and puts. Basic ML models from [16, 17] were selected to create a simple average of predictions, a.k.a., ensemble model. As specific nature of ML model is not relevant for illustration purposes of this example, following models were arbitrarily selected:

- K-nearest neighbor (KNN)
- Gradient Boosting Trees (GBT)
- Support Vector Machines (SVM)
- Gaussian Processes Regression (GRP)

Ensemble of above ML models is used to predict weekly expected lowest and highest price of gold serving as strike prices put and call options in a collar, respectively. Using gold price simulation discussed in [9] and used in [14, 15, 20], gold price predictions and weekly rebalancing costs of the collar are graphed below:





Total cost of rebalancing replicating portfolio over one year period, i.e., total cost of collars is used as a strike price for real option of investing in gold production. Results shown in table below:

			w=0	*w=4*	*w=8*	*w=12*	*w=16*	...	*w=48*	*w=52*
										640.98
									550.02	***603.68***
									512.78	471.97
									405.00	***434.68***
									367.76	347.53
G	42.03								298.21	***310.23***
η	87.66								***260.98***	255.90
r	0.02								219.58	***218.60***
PV	44.74		***w=0***	***w=4***	***w=8***	***w=12***	***w=16***	...	***182.35***	188.43
							161.69	...	161.69	***151.13***
σ	55%					138.74	***124.90***	...	***124.45***	138.74
u	1.17				119.06	***102.03***	119.06	...	119.06	***101.45***
d	0.86			102.16	***82.47***	102.16	***82.30***	...	***81.82***	102.16
p*	0.47	***η***	87.66	***65.83***	87.66	***65.58***	87.66	...	87.66	***64.87***
q*	0.53	***H***	***51.82***	75.22	***51.46***	75.22	***51.13***	...	***50.43***	75.22
dt	0.08			***39.69***	64.55	***39.24***	64.55	...	64.55	***37.93***
					29.51	55.39	***28.94***	...	***27.31***	55.39
H	37.30					***21.07***	47.53	...	47.53	***18.10***
PV+RO	96.56						***14.25***	...	***10.29***	40.79
								...	35.00	***3.49***
									1.63	30.03
									25.77	***0.00***
									0.00	22.11
									18.98	***0.00***
									0.00	16.28
									13.97	***0.00***
									0.00	11.99
										0.00

Therefore, hedging with Collar using variable strike prices predicted by ML ensemble lower and upper bounds results into investment project's value of:

$$PV = DCF + Real\ Option\ Value = 44.74 + 51.82 = 96.56$$

This shows value-add of only 9.4 per gram compared to value-add of 13.42 per gram from protective put with variable strike price strategy implemented in [20].

Conclusion

This paper answered a question of whether short collar hedge is more advantageous to protective put strategy. Using example of Zarapkhana, the answer was found to be less advantageous, precisely, by providing 4.02 per gram smaller improvement in real options value. Several factors could be responsible

for inferior results despite using more sophisticated hedging strategy. Two factors can be outlined and developed in further research. One is improvement of ML predictions or ensemble creation process or both. The other is obtaining more data from Zarapkhana on maximum price it is willing to pay per oz. gold that also keeps them solvent. Such price would guarantee call option strike as the minimum between ML predicted upper gold price and acceptable gold price that ensures positive operating profit.

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Challenges Financing and Pricing of Medical Services in Healthcare.

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Abstract
In this paper we are presenting existing mismatch between reimbursement based of Universal Healthcare Program adopted rules and real expenses of medical services in hospitals. Using real option valuation model of IT investments we are showing reasonable amount of investment which could be spend in order to get exact expenses structure of medical services and avoid mismatch between expenses and its reimbursements.

Introduction

With valuation of IT investments can be related the following challenges: difficulties to measure IT systems benefits in monetary terms, problems to recognize benefits caused by IT systems, time lag between IT project implementation and possessed benefits, difficulties with successful implementation of project and usage of IT systems which depends on required structural changes in organizations [1].

Because of above mentioned challenges can be difficult to use traditional/classical project valuation methods. One of the approaches to valuate IT investment in this case can real option analysis (ROA) [2], [3]. ROA valuates strategic opportunities of IT systems. But not all of them can determined from the beginning. Therefor we are considering not overall IT investment valuation, but how much of invested amount can be justified by benefits of some opportunity from this system [4].

In the article we will consider one of the opportunities of IT system installation for Georgian healthcare organizations: because of IT systems usage all hospital expenses[1] can be recognized and classified. Knowing exact structure of hospital expenses take crucial role get appropriate financing for medical services from governmental and custom insurance organizations. In our example we are considering reimbursement system from universal healthcare program in Georgia.

Hospital Expense Allocation Strategies

During designing hospital is very important to forecast all types of expenses and allocate them in the service Charge Masters.

Service Charge Masters, or Charge Description Master (CDM), is a comprehensive listing of items billable to a patient or a patient's health insurance provider. It lists prices for any service or product a hospital offers. The Charge Master is a living set of data elements. It is a menu of anything and everything hospital can charge a patient.

Hospital expenses are divided into direct and indirect costs.

Direct Costs are:

- Salaries of performer doctors and nurses (in some cases).
- Medicines and Supplies.
- Diagnostic and Lab Tests.

Indirect Costs are:

- Utilities and other Expenses.
- Administration Salaries.
- Fixed Salaries of Medical and Non-Medical Staff.

Expenses are also divided into fixed and variable expenses. Variable expenses are issuing according to services rendered and fixed costs are permanent costs of the hospitals to maintain administrative operations and manage medical and non-medical staffing fixed and duty salaries.

Expense structure of the hospital is adjusted to the medical and non-medical operational strategies.

Below described expense structure of the hospital that can be envisioned as appropriate allocation of expenses.

[1] All data had taken from one of the hospitals from Georgia. Because of confidentiality we shall not indicate the hospital name.

Hospital Expense Structure (% From Revenues)	
Salaries	**45%**
Physician's Salaries	**25%**
Nursing Staff Salaries	**10%**
Admin Staff Salaries	**5%**
Other Staff Salaries	**4%**
Other Staff Benefits	**1%**
Drugs and Other Consumables	**23%**
Overhead Expenses	**12%**
Business Development Expenses	**1%**
Marketing and PR Expenses	**1%**
Contingency	**1%**
EBITDA	**17%**
Total Sum	**100%**

Before planning new services, hospital must create Charge-Masters.

Charge Master consists with clinical and non-clinical/administrative expenses.

Creation of Charge Masters consists with following 5 steps:

1. Clinical Information Collection
Creation clinical workgroup of specialty's leading specialists
Using Clinical Evidence-Based Guidelines
Determination of medicines and supplies; lab and diagnostic tests used during patient treatment.
2. Administrative Information Collection
Expense category and rate identification
Administrative salary rate adjustment
Building, medical and non-medical equipment expenses calculation and rate identification (calibration, maintenance, insurance, depreciation and amortization)
Tax rate identification
3. Profit Margin Determination
Profit margin identification
4. Charge Master Formatting
5. Charge Master approval

Main reason of Charge Master creation is to determine the real costs of services and to adjust market prices for provided services.

Main Challenges of healthcare service expense allocations in Georgia

Georgian healthcare system developed very fast from 2007 according to the Government healthcare development and privatization project and during this fast development, service prices were regulated by market according to the competitive environment. Georgian hospitals adjusted their service prices according to the local healthcare market's historical pricing strategy.

From 2012 Government established Universal Healthcare Program and all hospitals were required to participate in elective surgical reimbursement program with historical prices (within past 1 year), plus 10% of values. And most of the hospitals still participate in Universal Healthcare Program with same prices. Despite inflation indicators, hospitals cannot increase their service prices.

Most of the hospitals never did charge masters and never calculated real costs of the services provided by them.

Hospitals are preparing individual case's Charge Masters, that includes information about direct costs, that are real costs of the services, but indirect costs are allocated as a formal value, indicated in percentages (e.g. 25%) or numbers (200 Gel) (according to bed day costs).

Main payer for healthcare services is still Government's Universal Healthcare Program. So, 60-70% of hospital's revenues are coming from UHC sources, most hospitals are paying according to old prices, and they cannot update their prices for UHC reimbursement purposes. Even Government is trying to limit existing prices, for minimizing UHC budget.

So, 60-70% of hospital's revenues consist with UHC reimbursement volume, according to Inappropriate prices, hospitals' revenues are not covering properly their real expenses.

ROI Analysis of Expense structure of the hospital

As its already stated above, without exact calculations of the cost of medical services hospitals suffers from losses because of insufficient reimbursements based on financing rules of Universal Healthcare Program. This is one of the important reasons of IT system installation in the hospital which can identify all expenses of medical services and can suggest proper amount of reimbursement. So, we can consider exact expense structure determination with IT system as a strategic opportunity of hospitals. Therefore, in order to estimate the maximum amount of investment which can be justified with such type of strategic opportunity, we can conduct real option analysis and build the investment valuation model.

Regardless that most of the expenses of the hospital have exponential or beta distributions, differences between expenses and reimbursements from government according to Universal Healthcare Program are normally distributed with mean 136,000 and standard deviation 87,000. We have used Monte Carlo simulation to get annual differences between costs and reimbursements during 10 years. Converting amounts of differences into percent of changes we have gotten lognormally distributed present value during 10 years of differences. We have used 10,000 iterations for Monte Carlo simulation. The table below presents results from Monte Carlo simulation.

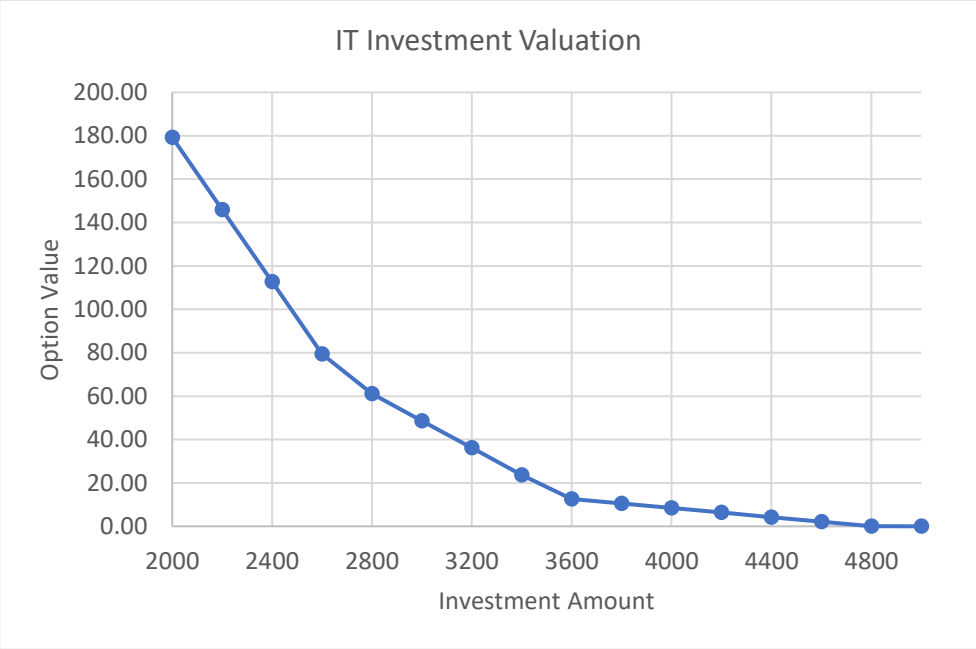
Amounts in (000)			Years	Cost Differences
Cost Differences	136		1	246.31
St.Dev.	87		2	166.51
Disc Rate	11%		3	239.22
			4	169.05
PV_0	1,091.63		5	34.06
PV_1	965.40		6	210.15
Z	-0.12288		7	182.46
			8	198.12
σ	0.1049		9	195.62
$E(PV_0)$	805.23		10	185.74
Confidence Interval (95%)				
Upper Bound	815.42			
Lower Bound	795.04			
St.Dev.	164.40			

Based of this model we have calculated real option value with Jarrow-Rudd binomial mode [5]. We cannot us most frequently used Cox Ross Rubinstein [6] model because lognormal standard deviation is less than discount rate – what violates the initial assumptions of Cox Ross Rubinstein binomial model.
The table below presents results of binomial valuation of the real option.

r	11%
σ	0.1049
Expacted Cost Difference	805.23
N	5
Δ	0.2
Investment	500
T	10
Up factor (u)	1.070
Down factor (d)	0.974
Risk-neutral measure (p)	0.5
Option Value	638.63

Using different amounts of investments as strike in real option model we can show the relationship between investment size and the amount of the real option.

Investment	Option Value
2000	179.27
2200	145.98
2400	112.69
2600	79.41
2800	61.13
3000	48.64
3200	36.16
3400	23.68
3600	12.57
3800	10.49
4000	8.41
4200	6.33
4400	4.25
4600	2.17
4800	0.09
5000	0.00



It can be easily seen that maximum amount of investment which can be justified by benefits received from identification of exact expenses of medical services by IT system is approximately 5,000,000 per 10 years.

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Integrated Management System in Education

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Abstract

Importance of integrated management systems (IMS) is growing more and more for organizations Interest in this subject indicates that IMS are seen as "management systems of the future". IMS is one of the most effective tool to lead effectively and make processes in the organization fluent. According to that, the aim of this article characterizes the possibility of building IMS through the identification of common elements and specific requirements in accordance with the ISO 9001, and ISO 45001 professional references. Part of the article is the methodology of building IMS in the educational organization. In order to achieve this aim, I will try to demonstrate the importance of integrated management system in frame of quality management and occupational health and safety in the educational organization.

Integrated Management System (IMS) includes all aspect of an organization's systems, processes and standards into one smart system. This system allows a business to have effective management tools, save time and increase efficiency. IMS is combination of all elements as a one whole system.

Keywords: Integrated Management System; PDCA cycle; ISO 9001; ISO 45001

Introduction

Integrated Management system in education combines integration of effective quality management system (QMS), occupational health and safety system (OHSAS) and information Technologies systems (ITS). Especially during the pandemic situation, when the whole educational system in the world had to turn into online, we obviously have seen the important of use technologies not only in teaching, but also in leading and management.

For Quality Management Systems, ISO 9001 is the most common and effective way to establish appropriate management system in the organization. ISO 9001 is defined as the international standard that specifies requirements for a quality management systems (QMS). Organizations use the standard to demonstrate the ability to consistently provide products and services that meet customer and regulatory requirements. ISO 9001 is based on PDCA cycle, which will be discussed later in this article and provides the organization all the necessary aspects for effective quality management systems.

As for the Occupational Health and Safety System, ISO 45001 is a way to establish the standard and make health and safety system procedures and processes effective in the organization. Unfortunately, not all

leaders understand the importance of this standard in education and mostly it is considered as a standard for construction or oil production companies. Later in the article we will see the importance of integration of both these standards for effective education management.

Finally, Information Technology Management, has become vitally important, especially while teaching and working online. It is important to combine Information Technology standard into the IMS in order to have effective human resources management. The most common and spread standard for implementation is ISO/IEC 20000-1:2018

PDCA Cycle in Integrated Management

H. James Harrington said: "Measurement is the first step that leads to control and eventually to improvement. If you can't measure something, you can't understand it. If you can't understand it, you can't control it. If you can't control it, you can't improve it." https://www.qualitydigest.com/may06/articles/02_article.shtml)

William Edward Deming, a prominent American researcher, similarly to Japanese, believed that management staff and all employees should be involved in the process of continuous improvement. He created 14 principles that later became the basis of the philosophy of quality in the organization and continuous improvement cycle PDCA (Plan - Do - Check - Act), called the Deming wheel. Deming cycle is a sequence of actions that aim at improvement. This cycle is also designed to solve quality problems and implement new solutions. PDCA model is extremely versatile and it can be successfully used in any type of business (https://deming.org/explore/pdsa/). The first "Plan" cycle in integrated management is one of the most important as far as it takes longer period than the other cycles and requires more work. "A man who does not think and plan long ahead will find trouble right at his door." – Confucius. In PDCA Cycle Plan includes such important business components as analyzing previous work with strong and weak sides; setting effective preventive actions; risk assessment; Design and revise business process components to improve results. So planning in IMS for both, Quality Management and Occupational Health and Safety directions should include all the components for effective planning. Planning in IMS of Education organization includes:

- Analyzing of the previous year's academic achievements
- Employees' evaluation results
- Customer satisfaction
- KPIs achievement
- Incidents and their root causes
- Non conformances and their root causes
- Risk assessment
- Customers and other interested parties' expectations

According to all mentioned above, the next step is making an effective plan, where all the employees will be involved and feel as part of the team. So, to sum up, the "Plan" cycle, it starts from analyzing the previous work in order to make an effective action plan. However, just because we made a good plan, does not mean that it will occur. Hence, the next step of Deming Cycle is "Do". Here the top management of the education organization needs to implement all the planned processes. Here is very important the term of team work as far as the teachers, lecturers, technical personnel and other employees should be involved in doing process. Otherwise, the aim of the organization won't be achieved as effectively as with their involvement.

Every process in business should be studied or checked. Within PDCA (Plan – do-check-act) Cycle Deming also uses PDSA (Plan-Do-Study-Act) cycle. In order to study or check, we should first have effective measurement tools. By this I mean, objective and reflective employees' observation forms; appropriate, customer satisfaction questionnaires, where both, quality and occupational health and safety standards requirements will be included. At stage of Check/Study, we should make a clear and obvious feedback as far as this stage is tightly connected with continuous improvement. Our Academic Personnel's professional development is based on effective evaluation system. At the same time, incidents, non-conformances, risks and near misses should be studied and investigated deeply in order to set effective preventive actions and avoid them in the future.

The Final stage of the Cycle is "Act", which includes taking actions based on the results of measurement. Setting effective actions in order to reduce the risks and avoid incidents and/or non conformances is a path to continuous improvement. Act is a part of the cycle, which analyses all other stages and leads to improvement. PDCA cycle and its implementation in integrated management system is given below on Figure 1.

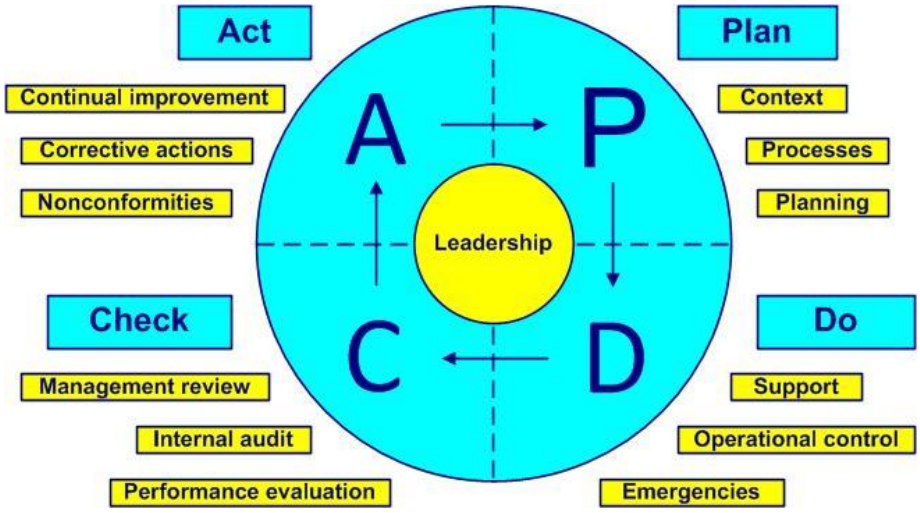
It is important to mention that in November 2021 there was made a small research in educational organizations (private and public schools and Universities of Tbilisi, Georgia). Participants of the survey were teachers/lecturers, top managers and customers (parents and students). According to the research, 100% of interviewed educational organizations have an action plan and they are aware about it. However, 75 % of the public sector and 40% of the private sector mentions that the plan is not based on the analyses of the previous academic year and/or employees do not have any information about last academic year's achievements, strengths and weaknesses. It may lead us doubt whether the plan can be effective and oriented on continuous improvement. Moreover, there is another question about the consideration of interested parties' expectations. Most of interviewed customers say that they have an access to the calendar of the educational organization, however, they do not have any information about the achievements, strengths and weaknesses of their institution.

94.4% or interviewed employees notice that they have systematic observations and receive the observation feedback on time. However, the result of Teachers' Subject Area Examinations according to the statistics of Ministry of Education and Science of Georgia, does not even exceed 25%. (https://edu.aris.ge/news/ramdenma-pedagogma-gadalaxa-minimaluri-zgvari-ratom-uchirs-naeks-am-informaciis-damushaveba.html)

As for the Occupational Health and Safety, unfortunately, there are no schools or Universities in South Caucasus with this Standard, while top Universities in the United Kingdom, US and Western European countries are proud to have the standard and effective integrated management system of ISO 9001 and ISO 45001. In list of these educational organizations are such top institutions as Boston College in the UK, University of South Carolina, Imperial College of London, University of Warwick and others.

To sum up, PDCA or Deming Cycle is a method of leading the educational organization effectively, however, in case of implementation of PDCA in integrated management system (IMS) is much more beneficial for business operation and managing educational organization, which finally should lead to good educational system in the country.

Figure 1



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Quality Management in Integrated Management System of Education

Quality Management means that the organization's culture is defined by and support the constant attainment of customer satisfaction through an integrated system of tools, techniques, and training. This involves the continuous improvement of organizational processes, resulting in high quality products and services. According to Deming, a system of quality improvement is helpful to anyone who turns out a product or is engaged in service, or in research and wishes to improve the output of the organization. The industrial analogy that compares workers and managers to students and teachers/lecturers is accurate and appropriate. In schools. Students are the workers and products. Teachers and administrators are managers. The hierarchy may look like this: students are the workers and the products. The difference between success and failure of the educational organization depends on the quality of their work. Teachers are the first level managers. Therefore the teacher will be leader of the class, emphasizing quality through non-coercive management featuring student as worker and teacher as coach, provoking the student to learn how to learn and thus to teach themselves. Heads of Departments/Deans are the middle and upper level management. The productivity of any educational institution depends mostly on the skills of those who directly manage the workers, i.e. the teachers/lecturers. According to Deming, their success in turn depends on how well they are managed by the administration above them. Therefore, any attempt at educational quality are best centered around organizational improvement efforts. The Board of Education is the board of directors thus responsible directly to the clients, and board members are overseers of the administration.

Management by Result is no longer sufficient to deal with the problems educational organizations are facing. In order to promote quality management, there is a need to change management philosophy. The new management philosophy focuses on achieving quality, which is defined as meeting and exceeding the needs and expectations of clients. A second focus is on the acceptance and pursuit of continuous improvement as the only useful standard or goal. The philosophy holds that example and experience teach little about theory, and that experience is not always useful knowledge. However, the philosophy is based on the acquisition and application of knowledge. This knowledge referred to as profound knowledge.

In order to provide leadership for quality management and mostly for integrated management, people in leadership must be able to understand and apply these concepts:

Systematic Thinking – this is the interdependence of functions with their sub-processes and of the organization with its people.

Theory of variation – this is the understanding of the difference between common and special causes. An understanding of variation will enable educational leaders to work toward quality within the framework of individual differences. The existence of variation is why a state of zero defects does not occur and why numerical goals are not feasible.

Theory of Knowledge – only through a theory of knowledge can one understand the past and predict the future. A major component of total quality management is prediction. Only through prediction and long-term perspective can educational organizations expect to succeed over a long period of time.

Knowledge of Psychology – the new philosophy is based on the understanding of people and their differences, and a commitment to applying systematic thinking to the people system. School leadership's aim is to free-up the potential of the different attributes of the people of the organization.

Quality comes not from inspection but from improvements of the process. In education, teachers/lecturers need to involve the student as a worker to evaluate the quality of his/her work, product or outcome. When students buy into the self-evaluation process the quality of their work is greatly enhanced. Using reality therapy techniques to find out what students want and what they are doing to get what they want sets the stage for this process of self-evaluation.

Occupational Health and Safety System in Education

The integration of occupational safety and health (OSH) into the educational system is an essential aspect of the development of risk prevention culture. This allows everybody, teachers/lecturers, students and parents alike, to learn how to live and work in a safe and healthy environment. The educational staff must be aware of the risk factors in their working environment, must realize the importance of accurate investigation of any incidents and near misses. They must also become acquainted with the legal regulations on safety and health at work in order to prevent accidents at the workplace. An educational institution must be a safe and healthy working environment for all the staff, students and other persons involved with it to make it suitable for the teaching and learning process. Implementation of Occupational Health and Safety into the integrated management system of education is not as simple as it may seem. First of all, it is mostly connected with educational organization staff's philosophy and mentality. Employees and top managers should feel and understand the importance of integrated management system within OHS as a path to effective management system. A very good and effective way to implement OHS Standard is ISO 45001.

In order to ensure occupational health and safety in educational institutions managers must implement an occupational health and safety system. This should be part of the overall management system and include the following elements:

- development of an occupational health and safety policy
- a management system which clearly allocates responsibilities in the field of occupational health and safety
- a risk assessment of health and safety at the workplace to be reviewed whenever conditions change
- occupational health and safety auditing
- training, information and instruction on health and safety at work
- emergency procedures -periodical analysis of the system in order to ensure that it is efficient
- storage of documentation and records in order to ensure continuity

OHS system in educational organizations should include the following directions:

Medical service. It is one of the most important parts of the system, which consists of doctor and phycologist's work. When we talk about educational institutions, medical service is vitally important for schools, as The school age is regarded as the most important phase of childhood life during which the child enters the society training system and emerges as a contributing member of the community. If the child doesn't maintain adequate health, the benefits of education will be lost because of absenteeism or lack of attention due to ill health and consequently a poor academic performance.

School health services deal with health appraisals, control of communicable diseases, record keeping and supervision of the health of school children and personnel. This aspect concerns itself with the evaluating the health of an individual objectively. Healthcare service gives the school authorities the opportunity to detect signs and symptoms of common diseases as well as signs of emotional disturbances that could impede the learning activities of children. Psychologist should have a big role in the process of observing students and employees psych-climate, evaluate students and employees' mental health and give effective consultations to both students and employees. There are a lot of fields, which must be studied by the educational organization's psychologist, from students and teachers/lecturers relationship to psych-climate between the employees inside the institution. Another very important object for psychological service is parents of students in schools. Unfortunately, a lot of parents have wrong attitudes towards children's raising methods and psychologist should be an intermediary link to build a correct relationship between parents and children.

Health services are both preventive and curative services and it helps in providing information to parents and school personnel on the health status of school children. It also provides advisory and counselling services for the school community and parents. It includes pre-entry medical screening, routine health screening/examination, school health records, sick bay, first aid and referral services. Other services rendered include health observation (which involves physical inspection of the physiology and behaviours of students/children), health examinations (screening tests and medical diagnosis) and health records (keeping of records of the health histories of students/children)

Security Service. This is second biggest department in OHS for educational institutions. This service should work due to regulations for keeping safety in educational institutions. The aim of this service is to strengthen security and emergency preparedness best practices; reduce safety risks and liabilities; improve students and employees perception of safety. Security service should not only guarantee security in all aspects in educational organization, but also lead the process of incidents and near misses investigation, set together with other responsible employees effective preventive and corrective actions and conduct trainings for the employees and students in order to keep safety and make people understand the importance of it.

Logistics Service, which should be responsible for infrastructure, purchasing, transportation and other services.

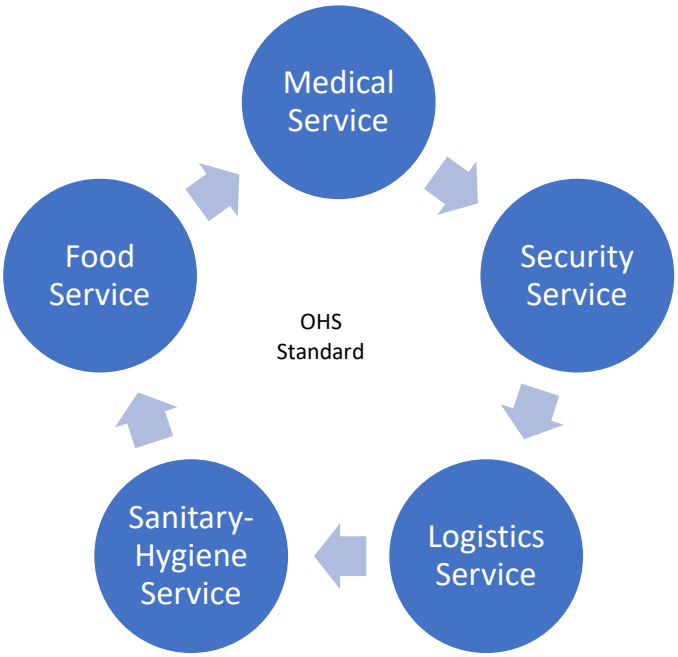
Sanitary – Hygiene Service, which is responsible for keeping the whole infrastructure clean following the regulations and norm of the standard and be involved in teaching students (in schools) how to keep and role of cleanliness of their own space.

Food Service, which is responsible for providing employees and students with healthy food. Usually the menu must be confirmed by the doctor and food manager.

These are the services, which make the occupational health and safety system work in the educational organization. However, it is important to remember that how these services work, the organization should decide and plan appropriately, make a clear policy, strategy and aim and follow the local regulations and norms of the Occupational Health and Safety Standard.

Figure 2 shows a clear connection of these services in Occupational Health and Safety system of educational organization.

Figure 2: Interrelated services in Occupational Health and Safety system of educational organization



Role of Technologies in Integrated Management System

Technology has revolutionized the field of education. The COVID-19 pandemic is quickly demonstrating why online education should be a vital part of teaching and learning. By integrating technology into existing curricula, as opposed to using it solely as a crisis-management tool, teachers can harness online learning as a powerful educational tool.

Technologies have been used as a major extent in governance and administration of educational institutions. Role of ICT has become one of the biggest in effective management of educational institutions.

Educational governance today increasingly needs to be understood as *digital educational governance*. The monitoring and management of educational systems, institutions and individuals is taking place through digital systems that are normally considered part of the backdrop to conventional policy instruments and techniques of government; technical systems that are brought into being and made operational by certain kinds of actors and organizations, and that are imbued with aims to shape the actions of human actors distributed across education systems and institutions.

In internal administration, the use of technologies has been recognized on a comprehensive scale. Educational administration is the process, by which methods, principles and procedures are put into practice within the educational institutions. It is vital for the individuals to carry out these functions in accordance to the goals and objectives. When the individuals are carrying out the governance and administrative functions, they need to ensure that they are able to achieve academic goals effectively. (Oyedemi, 2015). Today technologies in managing educational institutions can be used not only as a way of effective communication, but also correct time management, effective planning and decision making and objective measurement and monitoring tool.

Nowadays, in the era of timeless, effective and fast communication is one of the most important in management. People should have a free and fast access to necessary information. The communication processes between the individuals within the working environment is an easy and less time-consuming process. The individuals are able to access various forms of technology. In other words, connectivity is promoted among departments through technology and they are required to work in greater collaboration and integration. Through the use of technology and internet, the individuals are able to acquire information and augment their understanding in terms of concepts and fields. It facilitates organizational learning and adaptation to the changing global environment by the way of partnership, participation, information sharing and delegation.

To sum up, there is a need for new approaches for enhancing education for sustainable development in universities and schools. Implementation of integrated management system in educational institutions provides an active, safe and healthy environment toward sustainable development and it also causes increase in their quality levels. Implementation of IMS not only causes continual improvement, but it also familiarizes the public with new management systems, which would be a good pattern for using efficient management and policies.

Resources:

Jeremy Weinstein, Steve Vasovski, pp 5-11; the PDCA Continuous Improvement Cycle Module 6.4

Bluebird Holding Limited Integrated Management System; pp 12-16; 22-25; 44-48; 58-60; Version 01

Scott Eacott; A theory and Methodology for educational leadership, Management and Administration, pp 33-35; 67-70; 115

Harry Tomlinson; educational Leadership; pp 11-13; 17-19

https://www.iso.org/standard/70636.html

Semimartingale Convergence Sets

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Abstract

Semimartingale convergence sets are described in terms of predictable characteristics of semimartingale. The results play key role to study strong consistency of solution of Robbins–Monro type SDE (general recursive procedure) and special cases of continuous and discrete time models.

Key words and phrases: semimartingale, convergence sets, predictable characteristics.

MSC 2010: 62L20, 60H10

Let stochastic basis $(\Omega, \mathcal{F}, F = (\mathcal{F}_t)_{t\geq 0}, P)$ be given.

Let $X_\infty = \lim_{t\to\infty} X_t$ and let $\{X \to\}$ denote a set, where X_∞ exists and is a finite random variable.

Denote by $\mathcal{V}^+$ $(\mathcal{V})$ a set of processes $A = (A_t)_{t\geq 0}$, $A_0 = 0$, $A \in F \cap D$ (i.e., a process A is F-adapted with cadlag trajectories) with nondecreasing (bounded variation on each interval $[0, t]$) trajectories. We write $X \in \mathcal{P}$ if X is a predictable process. Denote by S_P a class of special semimartingales, i.e., $X \in S_P$ if $X \in F \cap D$ and

$$X = X_0 + A + M,$$

where $A \in \mathcal{V} \cap \mathcal{P}$, $M \in \mathcal{M}_{loc}$.

If $\Gamma_1, \Gamma_2 \in \mathcal{F}$, then $\Gamma_1 = \Gamma_2$ (P-a.s.) or $\Gamma_1 \subseteq \Gamma_2$ (P-a.s.) mean that $P(\Gamma_1 \Delta \Gamma_2) = 0$ or $P(\Gamma_1 \cap (\Omega \setminus \Gamma_2)) = 0$, respectively, where Δ is the sign of symmetric difference of sets.

Let $X \in S_P$. Put $A = A^1 - A^2$, where $A_1, A_2 \in \mathcal{V}^+ \cap \mathcal{P}$. Denote

$$\widehat{A} = (1 + X_- + A^2_-)^{-1} \circ A^1 \quad \left(:= \int_0^{\bullet} (1 + X_{s-} + A^2_{s-})^{-1} dA^1_s \right).$$

Theorem 1. *Let $X \in S_P$, $X \geq 0$. Then*

$$\left\{ \widehat{A}_\infty < \infty \right\} \subseteq \{X \to\} \cap \left\{ A^2_\infty < \infty \right\} \ (P\text{-a.s.}).$$

Corollary 1.

$$\left\{ A^1_\infty < \infty \right\} = \left\{ (1 + X_-)^{-1} \circ A^1_\infty < \infty \right\} = \left\{ \widehat{A}_\infty < \infty \right\} \ (P\text{-a.s.}).$$

The theorems below have been proved in [1].

Introduce the assumptions:

1) $EX_0 < \infty$;

2) one of the conditions (α) or (β) is satisfied,

   (α) there exists $\varepsilon > 0$ such that $A^1_{t+\varepsilon} \in \mathcal{F}_t$ for all $t > 0$,

   (β) for any predictable Markov moment σ,

$$E \, \Delta A^1_\sigma \, I_{\{\sigma < \infty\}} < \infty.$$

Theorem (A). *Let $X \in S_P$, $X \geq 0$, $X = X_0 + A^1 - A^2 + M$, $A^1, A^2 \in \mathcal{V}^+ \cap \mathcal{P}$, $M \in \mathcal{M}_{loc}$ and assumptions 1) and 2) are satisfied. Then*

$$\left\{ A^1_\infty < \infty \right\} \subseteq \{X \to\} \cap \left\{ A^2_\infty < \infty \right\} \ (P\text{-a.s.}).$$

Theorem (B). *Let $X \in S_P$, $X \geq 0$, $X = X_0 + X_- \circ B + A^1 - A^2 + M$, $B, A^1, A^2 \in \mathcal{V}^+ \cap \mathcal{P}$, $M \in \mathcal{M}_{loc}$ and assumptions 1) and 2) (for A^1 only) are satisfied. Then*

$$\left\{ A^1_\infty < \infty \right\} \cap \{B_\infty < \infty\} \subseteq \{X \to\} \cap \left\{ A^2_\infty < \infty \right\} \ (P\text{-a.s.}).$$

Theorem (B) follows from Theorem (A) if we apply the latter to the process $X \cdot \mathcal{E}^{-1}(B)$, where $\mathcal{E}(B)$ is the Dolean exponential, i.e., the solution of the equation $Y = 1 + Y_- \circ B$.

Now we reject assumptions 1) and 2) in both theorems based only on these theorems. Indeed, consider the process $Y = 1 + X$. Evidently, $\{X \to\} = \{Y \to\}$ (P-a.s.). Introduce the process

$$\widetilde{A} = \frac{1}{Y_-} \circ A^1 \in \mathcal{V}^+ \cap \mathcal{P}.$$

We have

$$\begin{aligned} X = X_0 + A^1 - A^2 + M \implies & Y = Y_0 + A^1 - A^2 + M \\ \equiv & Y_0 + Y_- \circ \widetilde{A} - A^2 + M, \end{aligned}$$

i.e., we get the decomposition of the process Y with $A^1 = 0$ and $B = \widetilde{A}$ (see Theorem (B)).

Now Theorem (B) gives that only under assumption 1)

$$\begin{aligned} \left\{\widetilde{A}_\infty < \infty\right\} & \subseteq \{Y \to\} \cap \left\{A^2_\infty < \infty\right\} \\ & = \{X \to\} \cap \left\{A^2_\infty < \infty\right\} \ (P\text{-a.s.}). \end{aligned}$$

But $\left\{\widetilde{A}_\infty < \infty\right\} = \left\{A^1_\infty < \infty\right\}$ (P-a.s.), since $\left\{A^1_\infty < \infty\right\} \subseteq \left\{\widetilde{A}_\infty < \infty\right\}$ (P-a.s.) and

$$\begin{aligned} & \left\{A^1_\infty < \infty\right\} \cap \{X \to\} \cap \left\{A^2_\infty < \infty\right\} \\ = & \left\{\widetilde{A}_\infty < \infty\right\} \cap \{X \to\} \cap \left\{A^2_\infty < \infty\right\} \ (P\text{-a.s.}). \end{aligned}$$

So we get that the assertion of Theorem (A) is true without assumption 2). Applying just proved fact to $X \cdot \mathcal{E}^{-1}(B)$, we get that the assertion of Theorem (B) is also true without assumption 2).

Further, denote

$$X^l = X_0 \wedge l + A^1 - A^2 + M,$$

for each constant $l > 0$. Evidently, $EX^l_0 = E(X_0 \wedge l) \leq l$.

Then by Theorem (A),

$$\left\{A^1_\infty < \infty\right\} \subseteq \{X^l \to\} \cap \left\{A^2_\infty < \infty\right\} \ (P\text{-a.s.}).$$

Hence $\{A^1_\infty < \infty\} \subseteq \{A^2_\infty < \infty\}$ and

$$\begin{aligned}\{A^1_\infty < \infty\} &\subseteq \{X^l \to\} = \{X^l \to\} \cap \big(\{X = X^l\} + \{X \neq X^l\}\big) \\ &= \{X \to\} \cap \{X = X^l\} + \{X^l \to\} \cap \{X \neq X^l\} \\ &\subseteq \{X \to\} \cap \{X_0 \leq l\} + \{X_0 > l\} \ \ (P\text{-a.s.})\end{aligned}$$

for each $l > 0$. Note, now, that $\{X_0 \leq l\} \uparrow \Omega$ (P-a.s.), and $\{X_0 > l\} \downarrow \emptyset$ (P-a.s.) as $l \to \infty$.

Thus

$$\begin{aligned}\{A^1_\infty < \infty\} &\subseteq \{X \to\} \cap \lim_{l\to\infty}\{X_0 \leq l\} + \lim_{l\to\infty}\{X_0 > l\} \\ &= \{X \to\} \cap \Omega + \varnothing = \{X \to\} \ \ (P\text{-a.s.}).\end{aligned}$$

Hence, we conclude that the assertion of Theorem (A) is true without assertion 1) as well.

Corollary 2. *Assertions of Theorem* (A) *and Theorem* (B) *are true without assumptions* 1) *and* 2).

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Organizational Triggers in Relation to Service Employees Performance

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ABSTRACT

In recent years, performance measurement has become a major issue. If an organization has the ability to measure its vital activities, and this at all organizational levels, then it will become critical to its success in today's fast-paced world. In complex and dynamic environment, for assessing the amount of desirability and utility of their activities each organization requires ranking and determining key performance indicators. They provide links among execution, strategy and ultimate value creation. Several studies have been conducted to explore the individual effects of organizational triggers on employees' performance. Nowadays in organizations there is a significant focus on services, creating the customer-responsive culture, and service standards. Based on an extensive literature in the field, the paper is about the relationship between organizational triggers and performance indicators for employees in the service field.

> *"Triggers is fantastic! We can't really reach our personal goals until we move away from self-centered goals. In order to become the person, we aspire to be, we need to embark on a journey of awareness that requires attention, action, and discipline."* (cited from (Goldsmith, 2015))
>
> David Chang, James Beard Foundation outstanding chef award winner, founder and CEO, Momofuku Group

Performance measurement is failing organizations, whether or not they are multinationals, government departments, or small local charities. The measures that are adopted were dreamed up sooner or later with none linkage to the critical success factors of the organizations. Usually organizations use the measurements on a monthly or quarterly basis.

Many companies are working with the incorrect measures, many of which are incorrectly termed key performance indicators (KPIs). The monitoring of true KPIs is a challenging goal for organizations. The four types of performance measures are (Parmenter, 2010):

1. Key result indicators (KRIs) tell you ways you have got in critical success factor.

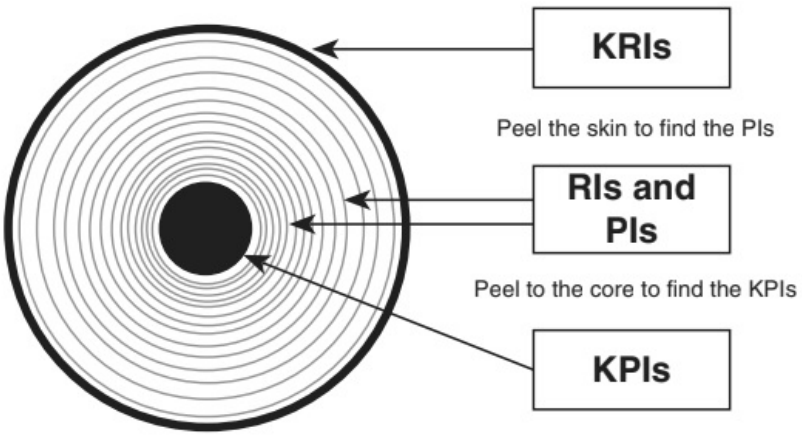
2. Result indicators (RIs) tell you what you have got done.

3. Performance indicators (PIs) tell you what to try to do.

4. KPIs tell you what to try to extent performance dramatically.

Many performance measures utilized by organizations are thus an inappropriate mixture of these four types. An onion analogy may be used to describe the connection of those four measures (see Exhibit 1). The surface skin describes the condition of the onion, the effect of sun, water, and nutrients it's received; and the way it's been handled from harvest to the supermarket shelf. Key result indicator is the outside skin. However, as we peel the layers off the onion, we discover more information. The layers represent the varied performance and result indicators, and therefore the core represents the key performance indicator.

Exhibit 1: KPIs system Analogy



Parmenter, D. (2010). *Key Performance Indicators.* John Wiley & Sons, Inc.

KPIs are powerful metrics which will provide insight into how your company is doing. You'll analyze several different criteria and find a pulse on multiple aspects of your operation. When you're ready to evaluate and analyze the performance of your company, it becomes easier to spot what's and isn't working. From here, you will target on your strengths and understand your weaknesses. To help you understand why KPIs are so important, you must know what other purposes they serve.

There are many reasons, but here are four that stand out to me:

- KPIs strengthen employee morale

I think it's important to start out with this value of a KPI because it's the less known. A company's culture is extremely important for performance. A culture that supports and motivates all of these in it's destined to try to be better than one that doesn't. In this sense, tracking KPIs could also be about acknowledging employees' labor and securing their feeling of accountability and responsibility. At our company, everyone has KPIs that they're responsible for. After we hit those numbers there's a way of ownership in our work and recognizable evidence of our contribution to the team. As an organization grows, sometimes there are often an increasing sense of distance between the organization's achievements and therefore the individual's efforts toward them. When people feel that they are responsible for KPIs, they're more likely to push themselves and receive more satisfaction from employment well done.

- KPIs influence and support business objectives

KPIs are important to business objectives because they keep objectives at the forefront of deciding. It's essential that business objectives are well communicated across a company, so when people know and are accountable for their own KPIs, it ensures that the business's overarching goals are top of mind. KPIs also make sure that performance is measured not blindly in pursuit of the KPI

but in respect to the larger business objectives. This implies that each part of work is completed with intentionality and for the correct purpose.

- KPIs foster personal growth

Not every product update or campaign will reach their targets. But when you monitor performance against those targets, it creates an environment of learning. With KPIs, teams are able to see exactly how they're engaging at any given moment. When you track KPIs, especially once you do so on a real-time KPI dashboard, you're able to ask what, why, how and when... and do so whenever. This makes to learn from successes and failures a daily activity. Another reason why KPIs are important for private growth builds off the concept of increased morale. Allowing employees to observe their performance and respond within the moment implies that they are more likely to attain their goals and better understand a way to do so within the future. This sense of continuous improvement allows people to realize much more than they may think, which is important for workplace satisfaction and continued personal growth.

- KPIs are critical for performance management

I'd say this last one is that the definitive reason why key performance indicators are important. It sums up all the reasons above: what's measured is managed. Employee morale, capacity and culture, among others, all contribute to performance. KPIs simplify performance management by allowing everyone to not only see what they're doing, but what others do in addition.

A trigger is something that performs a specific action. Also any stimulus that shapes our thoughts and actions can appear suddenly and it can be major or minor moments, pleasant or ambitious. Our environment is the most powerful mechanism of use in our lives, which does not always work in our favor, we make plans to achieve our goals, but the environment is constantly interfering. For example, when the smell of bacon comes out of the kitchen, we forget the doctor's

advice about lowering our cholesterol, and sometimes we have to skip a child's football game because we are obligated to appreciate the staff and provide them consistently because they work late every day and so on. This is how our environment interferes with our lives and we have to act against our will.

Our environment has feedback as a trigger. After all, our environment is constantly giving us new information that is important to our lives and changing our behavior. But the similarities end here. Our environment often leads to bad behavior and this is done against our will and against better judgment and without awareness, where a well-designed feedback loop leads to the desired behavior. Change is taking place.

Dr. Robert Cialdini is a famous author of the bestseller books about influence, he is widely known for his breakthrough ideas in the field of influence and power research, widely regarded as the 'Godfather of influence', because of his years of scientific research on the psychology of influence. Cialdini describes six triggers as the main drivers to influence others (Cialdini, 2007): reciprocity, scarcity, consistency and commitment, authority, liking, social proof.

Reciprocity is a deeply ingrained psychological trigger. If we act in favor of another person, they will have an obligation to return. In other words: benefits for benefits. The initial small kindness can lead to much greater feedback.

Scarcity is about the shortages. Something is difficult to achieve when there is a shortage of resources or the faster we want to achieve it. In short it is - scarcity. From an academic point of view, scarcity means that society does not have enough productive resources to meet all needs.

Consistency and Commitment - these are operated at the two levels. The first is that the best predictor of future behavior is its past. People try to conform to previous thoughts and actions. Therefore, making significant changes in lifestyle becomes much more successful when the goal is publicly announced. Second, the premise of great consent is small consent. In other words, the first "yes" to face-to-face sales is simpler than the next.

Authority is one of the scariest psychological triggers. Where social evidence is based on popular power - i.e., people 'just like me', - authority takes a principled step to use the power of particular individuals. Authority is dangerous because it has power. According to Cialdini, there is a tendency that the action may be repeated in response to simple symbols rather than its essence. Research has shown that the most effective symbol can be automobiles and clothes. In other words, people react not only to authority, but also to their appearance.

Liking is another obvious trigger that is not at all easy. No matter how logical our decisions may be, in reality "people prefer to give their yes to those who know and like them." In other words, if they like you they will buy from you. Approval can be expressed in both physical attractiveness and common interests and similarities.

Social proof - whether we like to admit it or not, the crowd is a powerful force. We can easily understand this if we consider sales. To increase sales, the first psychological trigger must come not from us, but from people who are already using the product or service.

Managers of organizations often talk about performance. There are objectives and targets to achieve – at a team and at an individual level – to meet organization's overall goals. Performance indicators as standards can express authority, as managers trigger their employees to achieve the standard. Managers must link employee's goals to business priorities, because when you build a foundation of trust in performance management it means that you are clear about what you expect from employees and specific about how their work ultimately fits into the larger picture of what the organization is trying to accomplish.

Working in service field receives increasing attention. Customer demands are changing rapidly, also the services that they require. Consumers participate within the service delivery process and services are provided from public stakeholder and therefore the funding is especially provided by public resources. Public services must refer to justice, fairness and equity, they're not only about efficiency and effectiveness.

Arguably, most people do not typically get up considering about their employment relationship. Even when nagging thoughts or perceptions exist, it typically takes something to really initiate

consideration of these thoughts or perceptions that prompt individuals to react. Louis and Sutton (1991) argued that some type of spark or trigger is required for people to maneuver from automatic mode to cognitively processing things at hand and making sense of it. Such cognitive processing is more active than simply noticing something, which is done in automatic mode. Accordingly, within the workplace, a particular event is required to trigger employees to maneuver from the automated, day-to-day processing described by Louis and Sutton (O'Neill & Cotton, 2017), to a more active consideration of an organization-related event.

In service industry, when conducting employee appraisals, managers search for competency in critical areas. In brief, managers want to know that employees are meeting established goals, working as contributing members of the team and applying critical thinking skills to assist business operations are successful.

Teamwork

Colleagues are generally seen as strong and contributing team players, when they work effectively together on group projects and initiatives. Examples, where employees exhibit a sense of team commitment include:

- Participating in group brainstorming.
- Volunteering for roles on team projects.
- Supporting others' ideas and approaches.

Communication

An important part of every service employee's job is an accurate, appropriate, professional sales communication. With the following in mind employers will evaluate this skill:

- Clear, concise verbal and written communications.
- Timely follow up to email and customer inquiries.
- An ability to accurately articulate ideas, concepts and feedback.

Customer Service

You are directly or indirectly serving your customer base through your position, regardless of the role you play. Employer will assess the employee in critical performance areas related to customer care, including:

- Professional, polite interactions with customers.
- Ensuring that problems are handled rather than being passed off.
- Offering options or solutions to resolve customer complaints.
- Timely responsiveness to customer needs.
- Perfect representation of the company.

Job Functions

Key performance indicators related directly to service employees' specific job functions will be appraised during an evaluation. Key performance indicators might include (Chron, a publication of Hearst Newspapers, LLC, 2020):

- Attention to detail.
- Timeliness.
- Good time management.
- Creativity and innovation.

I want to introduce the company where I work and talk about performance indicators in this public organization. Education Management Information System (EMIS) was established in 2012. The mission of the management system is to provide the education system with advanced technologies and electronic resources for the best education and management. The strategy of the Education Management Information System is to promote the functioning of the Ministry of Education and Science in the educational space through the introduction of modern information and communication technologies.

The KPIs aimed toward boosting the performance of the government officials in line with the government effort to boost public service delivery system and as assurance that the element of integrity and good governance are being carried out.

The study was conducted in EMIS using interviews with the respondents. The main objective was to study the implementation of key performance indicators in this government agency that offers services to the education system, to the universities, to the students...

- What are the main focuses of using KPIs in EMIS?

Key Performance Indicators at EMIS are focusing on two main aspects. EMIS develops the KPIs to measure efficiency and effectiveness of working process to supply and deliver services to customers (universities, schools, students). The efficiency of the core process to supply and deliver the services to customers is assessed in term of how accurate the service provided and delivered to customers. The work process is being assessed by looking at the statistics develop by the management team reckoning on their core business activities.

Triggers, or triggering events, are outlined as circumstances that act as catalysts to structure learning. Like persons, organizations don't learn proactively (Watkins & Marsick, 1993). Given the tremendous pressures to perform and turn out results, organizations tend to over-invest in exploiting existing data and under-invest in learning or developing new data (Levinthal, 1991). Moreover, in order for organizations to learn, people should learn. Individuals carry out what's expected of them, each written and unwritten expectations. Written expectations are usually delivered through job descriptions, memos, e-mails, and official documents. What's less clear for people within a structure are the unwritten expectations. For understanding unwritten expectations, there are three groupings in organizations: motivators, enablers and triggers, according to Maira Arun and Peter Scott-Morgan (Arun & Scott-Morgan, 1997).

Workplace is a stressful environment, which involves many situations and for that reason it may trigger strong negative feelings. It is important for managers not only to help workers control emotional situations but be able to control their own feelings. This can help contribute to a healthy environment, allow workers to perform according to their potential and maintain

workers' morale. It is important for most stressful situations that manager is able to respond in a rational, calm and positive manner. This helps encourage workers to see the situation more objectively. In contrast, when managers add their own emotions into the mix, it can be very unhelpful for fueling workers' emotions. Managers can send the message to workers, when they react in unhelpful ways, that they can lead the team through hard times and are incapable of remaining calm. When managers can help resolve an emotionally charged problem and demonstrate empathy can give workers confidence and that these workers are overseen by competent and strong leaders.

Qualitative and Quantitative researches were conducted in EMIS. I decided to make interviews with the people who were involved in managing people. Their opinion was interesting and important about organizational triggers. I was interested which organizational triggers were used in these companies and how these triggers were used by managers for improving the performance quality of service employees.

The results from the interview helped to define some basic organizational triggers, incorporated in the target organization. The most important part of the interview was about identifying the organizational triggers that were widely used in this organization and on which my further quantitative research should be based. From the listed organizational triggers, they mentioned seven triggers which they thought was widely used by their company:

- Scarcity
- Consistency and commitment
- Social proof
- New executives
- New service announcement
- Increase in expenses
- Awards

The second part of this study involves conducting quantitative research because it seeks to analyze how literature review and qualitative research relate to performance induced by organizational triggers in an organization.

Qualitative research lead to the defining of the seven triggers, considered by managers in target organization as widely used. Following the main goal of the study, for each of the trigger the following hypothesis is tested in the qualitative research: organizational trigger influences the performance.

Since the questionnaire assumes answers formed as Likert scale, we use Wilcoxon signed rank test for the paired samples of organizational trigger and performance to test the hypothesis about whether applying the given organizational trigger significantly influenced the performance. Normality of the sample observations is not assumed. The following hypothesis is formulated:

$$H_0: There\ is\ no\ difference\ between\ sample\ pairs\ (median\ change\ is\ zero)$$

$$H_1: There\ is\ a\ difference\ between\ sample\ pairs\ (median\ change\ is\ non-zero)$$

The procedure for testing the hypothesis is as follows. For n sample pairs (a total of 2n data points) and $i = 1, \dots, n$, let x_{1i} and x_{2i} denote the pair of respondent answers. The test statistic is defined as

$$W = \sum_{i=1}^{n} [sgn(x_{2i} - x_{1i})R_i]$$

Which can be regarded as the sum of signed ranks where sgn denotes the sign function and is defined as

$$sgn(y) = 1_{\{y>0\}}y - 1_{\{y<0\}}y,$$

and R_i denotes the rank. Under the null hypothesis, W follows a specific distribution with the expected value of 0 and a variance of

$$\frac{n_r(n_r+1)(2n_r+1)}{6}.$$

As long as the quantity W is computed, it is compared to a critical value from the Wilcoxon signed rank test table according to the sample size and the significance level α. We test all the hypothesis for $\alpha = 0.05$.

For the hypothesis testing in practical research (for each of the elaborated triggers it is that trigger significantly influences the performance) there was used a survey with the quantitative data analysis. Involvement of sample population was 63 employee.

Sample pairs that were used in hypothesis are:

Organizational trigger	Performance
Scarcity	Work done on time
Consistency and Commitment	Customer satisfaction
Social proof	Satisfaction of customer issues
New executives	Flexibility at work
New service announcement	Adjust changes in work
Increase in expenses	Taking responsibility
Awards	Quality of work

Very specific and realistic answers were the results of quantitative research. There was used sample pairs from organizational trigger and employee performance that were used then in hypothesis. Both hypotheses were tested statistically and at last generalized to the whole employed population.

Based on the results we conclude that the null hypotheses in all cases is rejected, which means that all organizational triggers – scarcity, consistency and commitment, social proof, new executives, new service announcement, increase in expenses and awards influence the employee performance. In particular, all of the organizational triggers changed employee performance significantly.

These findings scientifically prove that with organizational triggers managers can really manage service employees' performance. So a lot depends on them. By focusing on the development of employees and the alignment of company goals, managers can create a work environment that enables both employees and companies to thrive. In any organization, it's important to understand what your employees are doing, why they are doing and how they are doing it. When there is no system in place to define roles, provide constructive feedback, understand individual strengths and weaknesses, trigger interventions and reward positive behavior, is much more difficult for managers to effectively lead their employees.

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LIFE PERFORMANCE SCORE

IN MANAGING INTANGIBLE ELEMENTS IN ORGANIZATIONS

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ABSTRACT

Intangible (symbolic) elements in organizations, such as organizational culture, attitudes, emotions, relationships, creativity, success and values are vague and difficult to be measured and managed. However, managers deal with them at the daily basis, and these elements have significant impact on organizational performance. There is a need of a complex and multidimensional instrument development for monitoring 'how to be human at work'. In the paper it is described the planned study, survey development process and theoretical foundations of Life Performance Score instrument. Life Performance Score is performance in life, i.e. achieving the preferred results in different life areas continually in a way which is as effective, efficient and comfortable as possible for a person. It consists of 26 dimensions, measured in five basic criteria: noting; awareness and choice; actions; feelings; comfort.

***Key words**: life performance, intangible elements, organizational life, human at work*

> Successful organizations understand the importance of implementation, not just strategy, and moreover, recognize the crucial role of their people in this process.
>
> Jeffrey Pfeffer (Pfeffer, 1981)

Managers meet many challenges in managing the intangible (symbolic) elements in organizations, as attitudes, values, power, status in groups and others. These elements without objective foundation can be treated as phenomena, i.e. they are objects of sensory intuition, and can be objectified only in interpretation with the categories. The intangible elements are

based on the subjective interpretation, regardless to the systems created for their categorization. In managing the intangible elements in organizations there is a need of other managerial tools, different from the traditional planning, organizing, leading and controlling functions.

Basic clusters of intangible organizational elements are the following three: values (organizational culture), relationships, and creativity/success. Organizational culture as a symbolic system of values underlying the behaviors and norms in organizational life, appears from the collective psychological agreement on what is important and what matters. It is transmitted with the help of deep archetypes and symbolic mechanisms, so difficult to be monitored and managed. Relationships as another intangible element as well are quite vague to be traced, for example, in a group of two people there is one relationship and in a group of seven people already 21. Relationships have symbolic characteristics as they include building connections through communication, i.e. signs, images and metaphors. Creativity and success as a part of organizational life are connected with quite subjective processes of thinking, sensing and interpretations. Creativity is understood here broadly as the process of associating, emerging and thinking; success is perceived subjectively by people in organizations, and by organization itself.

Many experts nowadays try to find and offer the validated instruments for measuring, monitoring and managing intangible elements in organization. The following models and practical instruments are very popular in management: Emotional Intelligence (Daniel Goleman) for dealing with emotional part in work life, Organizational Culture levels and competitive values framework (Edgar Schein (Schein, 2010); Cameron and Quinn (Cameron & Quinn, 2011)), Person-Organization fit (Chris Argyris (Argyris & Schon, 1992), Jennifer Chatman (O'Reilly, 1991)), work-related attitudes (job satisfaction, job involvement, employee engagement, organizational commitment). If we assume the work life in organization as a social activity cycle, as a person joins organization and becomes employee, bringing the values

and goals, which in ideal should be aligned with the organization. Organizational culture and PO (person-organization) fit are significant instruments at this stage. Inside organization in adapting and socializing the employee starts building relationships with others, and again meets intangible element of organizational life. In this process the employee faces some problems, and some of them are unstructured, which demand non-programed decisions, here creativity comes to limelight. At work one can feel oneself more or less successful, and for managing success as an intangible element there can be used coaching. These elements, processes around them and tools for managing them are about one important issue for nowadays managerial focus, - How to be human at work.

In management, from one side, it is about how to be a human at work, from another, - how to integrate being human into organizational performance, and build advantage on that. Following mentioned above models and practical frameworks, we assume that there is some score which can help to measure and display, how much well a person does in life. In other words, what is the person's performance in life (i.e. achieving the preferred results in different life areas continually in a way which is as effective, efficient and comfortable as possible for a person). We call this indicator Life Performance Score.

In integrative paradigm we base this indicator on the 26 dimensions, which are integrated from the Embodied Yoga Principles (EYP) framework (Mark Walsh). These dimensions display most life challenges people have. There are added the poses or postures practiced in the Embodied Yoga for exploring and developing the desired emotional states or qualities. These postures practice we would offer to the participants of our study.

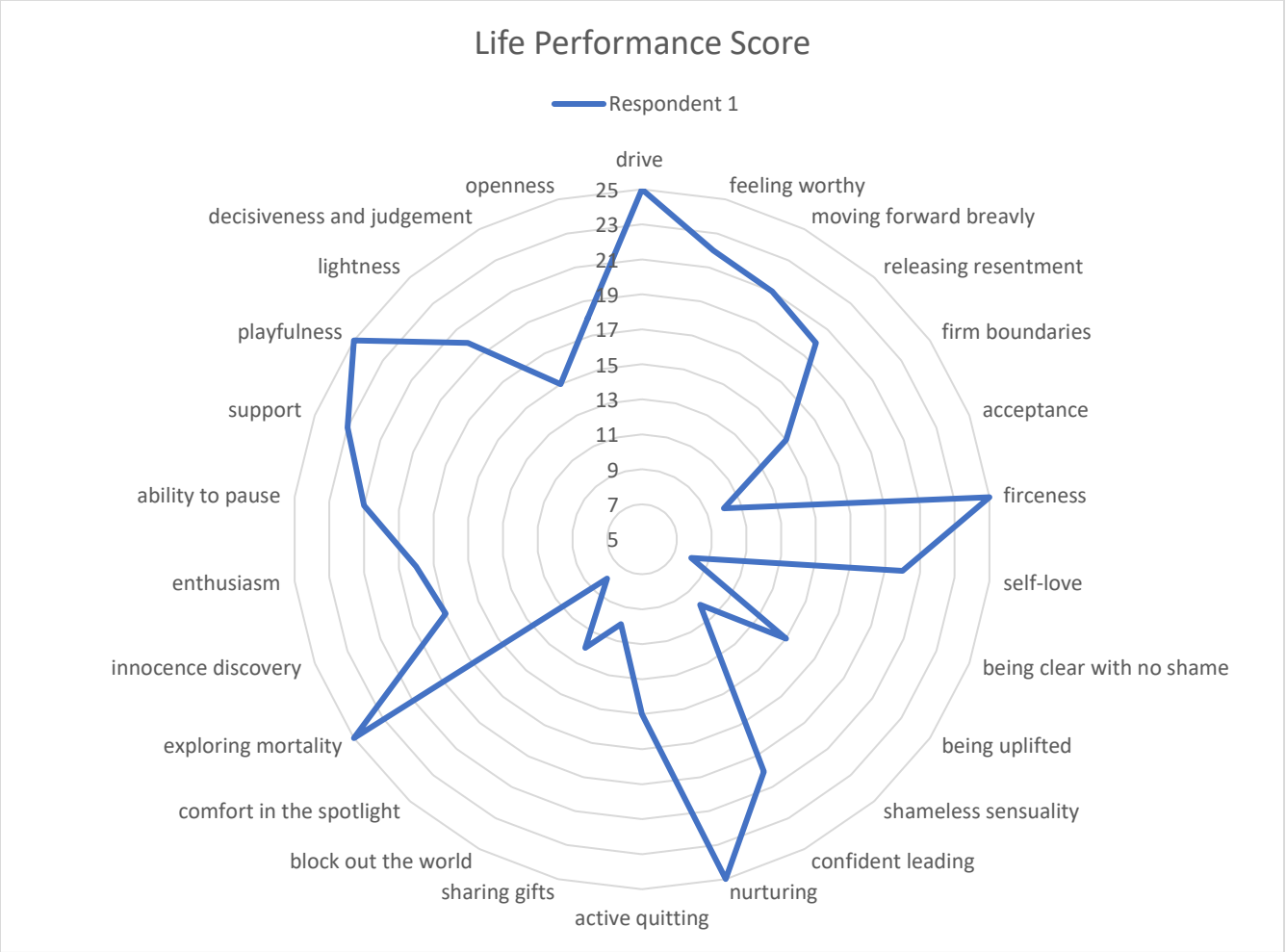
1. Drive (pushing pose)
2. Feeling worthy (receiving pose)
3. Moving forward bravely (entering pose)
4. Releasing resentment (letting go pose)

5. Firm boundaries (No pose)
6. Acceptance (Yes pose)
7. Fierceness (warrior pose)
8. Self-love (self-care pose)
9. Being clear with no shame (stand pose)
10. Being uplifted (inspiration pose)
11. Shameless erotic (sensuality pose)
12. Confident leading (authority pose)
13. Nurturing (care pose)
14. Active quitting (surrender pose)
15. Sharing gifts (giving pose)
16. Block out the world (inner pose)
17. Comfort in the spotlight (taking up space pose)
18. Exploring mortality (death pose)
19. Innocence discovery (vulnerability pose)
20. Enthusiasm (passion pose)
21. Ability to pause (rest pose)
22. Support (support pose)
23. Playfulness (Joker pose)
24. Lightness (transcendence pose)
25. Decisiveness and judgement (evaluation pose)
26. Openness (openness pose)

The study will lead to building an individual profile of Life Performance for respondents and then to check it again after practicing these 26 dimensions in embodied approach via postures. Exhibit 1 shows the example of the profile. Each of the dimensions (and its posture embodied

practice) are developed via multidisciplinary and holistic methods. In the foundations are archetypes, Eastern philosophy, different embodied practices (martial arts, yoga, dance), mindfulness, coaching principles. Embodiment is defined as "the manner by which (how) we (it is inter-relational) create (it is generative) our (identity is key) being (are), moment by moment in relationship to all our social, cultural and environmental contexts" (Walsh, 2021).

Exhibit 1



We assume that these dimensions reflect the main psychological, emotional and behavioral states, important for wellbeing in life. They can be displayed contingently, related to the need and choice in the situation. They constitute the potential (capacity) to make choices with

awareness, and to have the wider spectrum of options (in behavioral responses, dealing with environmental forces, and in relationships with people) in different life areas, including work life. The survey is based on the following structure of questions, where each dimension is measured in Likert scale 1-5 (1 fully disagree – 5 fully agree), and aligned with five criteria (the criteria are elaborated based on different scales used in psychology for measuring self-regulation, emotional intelligence, alexithymia, behaviors):

- noting
- awareness and choice
- actions
- feelings
- comfort

For example:

Drive (pushing pose) Pushing, pressure, to deal with that, to manage that. Exploring confidence, power, persistence and persuasion, moving forward, overcoming challenges, ambitions.

- How much I notice my feeling of confidence and power?
- How much aware I choose persistence regardless difficulties I face?
- How much confident I make steps to promote/push forward goals/projects/myself?
- I feel myself calm and confident in persuading and promoting my position.
- How much comfortable I feel in the state of power and promotion?

In the study to validate our developed survey we would use two more scales: Toronto Alexithymia Scale (TAS-20, consisting of 20 items, it is the instrument to measure alexithymia – trouble in identifying and describing emotions with the behavior of minimizing emotional inner experience and maximizing external attention focus) and Self-regulation behavior style Survey (it is the instrument for diagnosing self-regulation individually and its development, as planning, modelling, programming, evaluating, as well as personal qualities – flexibility and independence).

Life Performance Score measured and developed in a profile, would help to make 'being human' at work experience more measurable and clear, as well it would give the opportunity to explore its influence on organizational performance and its impact on intangible organizational elements.

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კიბერრისკი, როგორც სისტემური მნიშვნელობის საოპერაციო რისკი

ავტორი: დავით პაპუაშვილი

საოპერაციო რისკი 21 საუკუნის ერთერთ ძირითად რისკს წარმოადგენს. როგორც ახალი კორონავირუსით გამოწვეულმა გლობალურმა ეპიდემიამ წყადყო, საოპერაციო რისკისგან გამომდინარე მოვლენებმა, ხშირად მოულოდნელი და დიდი ზეგავლენა შეიძლება იქონიოს როგორც საზოგადოებაზე, ასევე ეკონომიკაზეც. ასეთ რისკის კატეგორიას მიეკუთვნება კიბერრისკიც, რომელიც ორგანიზაციების ციფრული ტრანსფორმაციის შედეგად, მზარდ მნიშვნელობას იძენს თანამედროვე საზოგადოებაში.

საოპერაციო რისკის ფართოდ აღიარებულ განმარტებას თუ მივყვებით, საოპერაციო რისკი წარმოადგენს დანაკარგის რისკს, რომელიც გამოწვეულია გაუმართავი ან ჩაშლილი შიდა პროცესებისგან, ადამიანებისგან, სისტემებისგან და გარეშე მოვლენებისგან. კიბერრისკი, რომელიც ხშირად ორგანიზაციის ელექტრონულ საინფორმაციო სისტემებსა და ზოგადად ელექტრონულ სივრცეს უკავშირდება, საოპერაციო რისკის განუყოფელი ნაწილია. შეიძლება ითქვას ისიც, რომ მიუხედავად კიბერრისკის ტექნოლოგიური ხასიათისა, კიბერრისკი ფინანსურ და ბიზნესს რისკს წარმოადგენს. შესაბამისად, კიბერრისკთან დაკავშირებულმა მოვლენებმა როგორც პოტენციური, ასევე, რეალური ფინანსური ზარალი შეიძლება მოუტანოს ორგანიზაციას, ასევე, საზოგადოების თითოეულ წევრსაც.

მიუხედავად იმისა, რომ კიბერრისკი საოპერაციო რისკის ერთერთ ნაირსახეობას წარმოადგენს, საოპერაციო რისკის სხვა ძირითად კატეგორიებთან შედარებით, კიბერრისკს გააჩნია მნიშვნელოვანი განმასხვავებელი მახასიათებლებიც. საოპერაციო რისკი ძირითადად იდიოსინკრატულია, რაც იმას გულისხმობს, რომ ის მოიცავს ან უკავშირდება ერთ ორგანიზაციას. შესაბამისად, საოპერაციო რისკი უმეტესწილად დაკავშირებულია ორგანიზაციის შიდა გარემოსთან და ისეთ პროცესებთან რაც ადამიანის ქმედებებიდან(მ.შ. შეცდომებთან) და დანერგილი საინფორმაციო ტექნოლოგიებს მოიცავს.

კიბერრისკი ამავდროულად არ არის იდიოსინკარტული ხასიათის რისკი. საყურადღებო განმასხვავებელი ფაქტორი არის კიბერრისკის გავრცელების სიჩქარეც. სხვა საოპერაციო რისკებისგან განსხვავებით, კიბერ-ინციდენტი სისტემაში შეიძლება გავრცელდეს უკიდურესად მოკლე ვადაში. მაგალითისთვის შეგვიძლია მოვიყვანოთ 2008 წლის კიბერშეტევები საქართველოზე, როდესაც ქვეყანა თითქმის მყისიერად მოწყდა გარე საინფორმაციო სამყაროს, ხელმიუწვდომელი იყო როგორც ფინანსური მომსახურება, ასევე ზუსტი და სწორი ინფორმაცია ქვეყანაში განვითარებულ მოვლენებთან დაკავშირებით. დამატებით შეგვიძლია მოვიყვანოთ 2017 წლის NotPetya-ს კიბერ-ინციდენტი უკრაინაში, როდესაც მსხვილ ბანკებზე გავრცელდა კიბერშეტევა 1 წუთზე ნაკლებ ვადაში. აქედან გამომდინარე, შეგვიძლია დავასკვნათ, რომ კიბერრისკის შემთხვევაში, გავრცელების მასშტაბი არ არის შეზღუდული გეოგრაფიული სივრცითა და გავრცელების შესაძლებლობით.

რა არის სისტემური კიბერრისკი?

სისტემური კიბერრისკი შეიძლება წარმოიშვას რამდენიმე სხვადასხვა მიზეზის გამო. ერთის მხრივ სისტემური რისკის ეს ფორმა წარმოადგენს ორგანიზაციისთვის თანმდევ რისკს, რომელიც უკავშირდება ციფრული ბიზნეს სისტემების სირთულესა და კომპლექსურობას. მაგალითად, როდესაც რამდენიმე ორგანიზაცია იყენებს მოძველებულ ელექტრონულ საინფორმაციო სისტემას, რომელიც სათანადოდ დაცული არ არის, ეს შეიძლება გახდეს ეკონომიკის რომელიმე სექტორისთვის განმაპირობებელი სისტემური კიბერრისკი. აღსანიშნავია ისიც, რომ სისტემური რისკი, რამდენიმე კომპანიაზე ან ეკონომიკის ერთ ან რამდენიმე სექტორზე თითქმის ერთდროულ ზეგავლენას ახდენს.

დამატებით, სისტემური კიბერრისკი შეიძლება გამომდინარეობდეს მესამე მხარესთან დაკავშირებული რისკიდან, რომელიც გადმოედინება რომელიმე კონკრეტულ ორგანიზაციაზე. აქ ძირითადად იგულისხმება აუთსორსინგის რისკი რაც უშუალოდ კომპანიების, ასევე ზოგადად ორგანიზაციების მიერ, გარეშე ელექტრონული საინფორმაციო სისტემების გამოყენებას უკავშირდება. ანალოგიურად რისკი შეიძლება იქიდანაც წარმოიშვას, როდესაც ერთი სექტორიდან ან თუნდაც კომპანიიდან სხვა სექტორზე ან ბიზნესზე გადაედინება კონკრეტული მოვლენა. მაგალითად, კიბერრისკი, რომელიც თავდაპირველად საინფორმაციო ტექნოლოგიურ სფეროს უკავშირდება, შეიძლება საკმაოდ სწრაფად გავრცელდეს ფინანსურ სფეროზე ან სადაზღვევო სფეროზეც, როდესაც ამ სფეროებში მოღვაწე კომპანიები ისეთ საინფორმაციო სისტემებსა და პროგრამულ უზრუნველყოფას იყენებენ, რომელიც შეიცავს კიბერრისკის ელემენტებს. აღსანიშნავია ისიც, რომ გარკვეულ შემთხვევებში, ციფრული ბიზნეს სისტემის შეფერხებას შეიძლება კასკადური ეფექტი ჰქონდეს სხვა სისტემებზე და პოტენციურად საზოგადოებაზე.

ცხრილი N1 ასახავს კიბერრისკის იმ ძირითად წყაროებს, მოტივაციასა და მიზნებს, რომლის შედეგად, ქვეყანაში შეიძლება წარმოიშვას კიბერრისკი.

ცხრილი 1: კიბერრისკი: წყარო, მოტივაცია და მიზნები

ძირითადი წყარო	მოტივაცია	მამოძრავებელი მიზნები	მაგალითები
სახელმწიფო, სახელმწიფოსთან დაკავშირებული ჯგუფები	გეოპოლიტიკური, იდეოლოგიური	მომსახურების შეფერხება, მონაცემთა განადგურება/მოპარვა, დაზვერვა, ფინანსური მოგება	• მონაცემების დაზიანება • ფიზიკური განადგურება • თაღლითობა • ელექტროსისტემების შეფერხება
კიბერკრიმინალები	პირადი გამდიდრება	პარვა/ფინანსური სარგებელი	• მონაცემების მოპარვა • თაღლითური ტრანზაქციები • ფულის მოპარვა
ჰაქტივისტები, ინსაიდერული საფრთხე, ტერორიზმი	იდეოლოგიური, უკმაყოფილება	მომსახურების შეფერხება	• მონაცემთა გაჟონვა • ვებ გვერდების სახის შეცვლა • მომსახურების შეფერხება

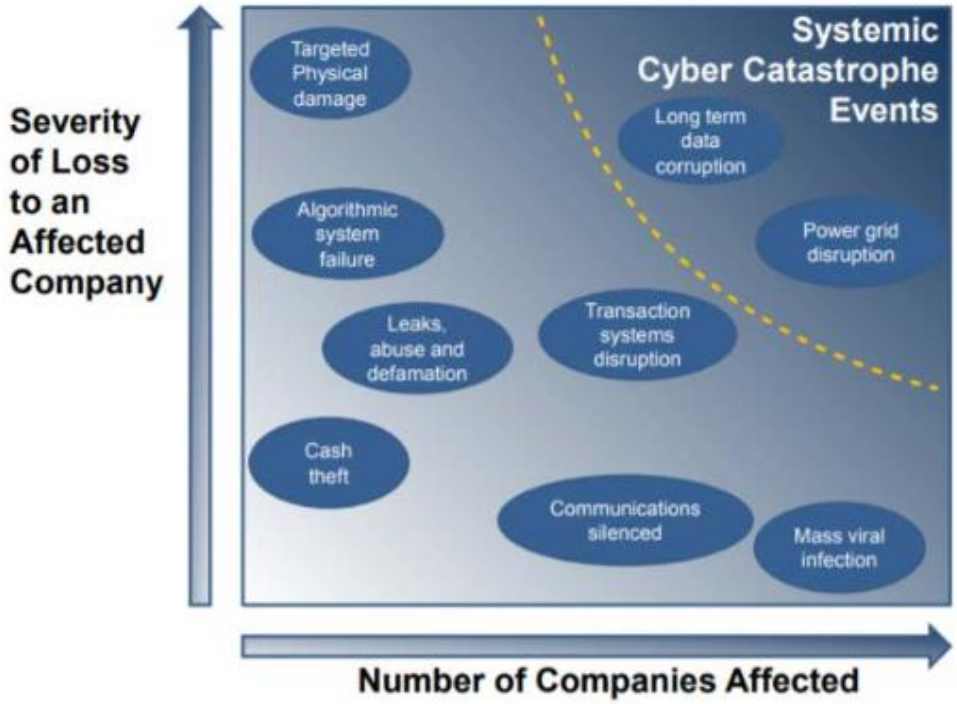
მოცემულ ცხრილში განხილულია კიბერრისკის სამი ძირითადი წყარო, თუმცა სისტემური რისკის თვალსაზრისით, რომელსაც შეუძლია ქვეყნის ეკონომიკის ერთ, ან რამდენიმე სექტორზე

მძლავრი ზეგალვენა იქონიოს, უფრო მეტად უკავშირდება გარკვეული სახელმწიფოებიდან, ან სახელმწიფოსთან დაკავშირებული ჯგუფების მიერ განხორციელებულ კიბერ-მოვლენებს. ასეთ შემთხვევებში, მოტივაცია შეიძლება იყოს როგორც გეოპოლიტიკური, ასევე იდელოგოიური. ამავდროულად, ხშირად ასეთი შეტევების მიზანს მომსახურების ფართომასშტაბიანი შეფერხება, სექტორისთვის მნიშვნელოვანი მონაცემების დაზიანება/განადგურება, ან ზოგ შემთხვევაში ფინანსური მოგება წარმოადგენს.

ანალოგიურად, დიაგრამა N1-ში ასახულია სხვადასხვა სახის კიბერ-ინციდნეტის მასშტაბი და სიდიდე. მიუხედავად იმისა, რომ კიბერრისკი სხვადასხვა ფორმას შეიძლება იღებდეს დაწყებული მონაცემთა მოპარვით, ტექნიკური გაუმართაობითა და თანხის მითვისებით, დამთავრებული მონაცემების გრძელვადიანი დაზიანებით, სისტემური რისკის თვალსაზრისით ყველაზე საყურადღებოა მოცემული დიაგრამის ზედა მარჯვენა კუთხე, სადაც თითოეული მოვლენის ზეგალენა საზოგადოებაზე მაღალი, ან უკიდურესად მაღალია და იმ ორგანიზაციების რაოდენობა, რომელსაც შეეხო კიბერშეტევა, ასევე დიდია.

დიაგრამა 1: კიბერინციდენტების მასშტაბი და სიდიდე.

წყარო: ევროპის სისტემური რისკის საბჭო და კემბრიჯის რისკის კვლევის ცენტრი



მაგალითისთვის, შეგვიძლია კვლავ დავუბრუნდეთ უკრაინაში 2017 წელს განხორციელებულ კიბერშეტევას, რომლის ფარგლებშიც ასობით კომპანიის საინფორმაციო სისტემა დაინფიცირდა მავნე პროგრამული კოდით და რომლის შედეგად კომპანიის მონაცემები და ინფორმაცია მნიშვნელოვნად დაზიანდა და სხვადასხვა სახით ხელმიუწვდომელი გახდა. მოცემული კიბერ-

ინციდენტის დროს, უკრაინის ენერგო-მატარებლებსაც შეუფერხდათ საქმიანობა. შესაბამისად, იმისათვის, რომ მოვლენა ჩაითვალოს სისტემური მნიშვნელობის ინციდენტად, ის:

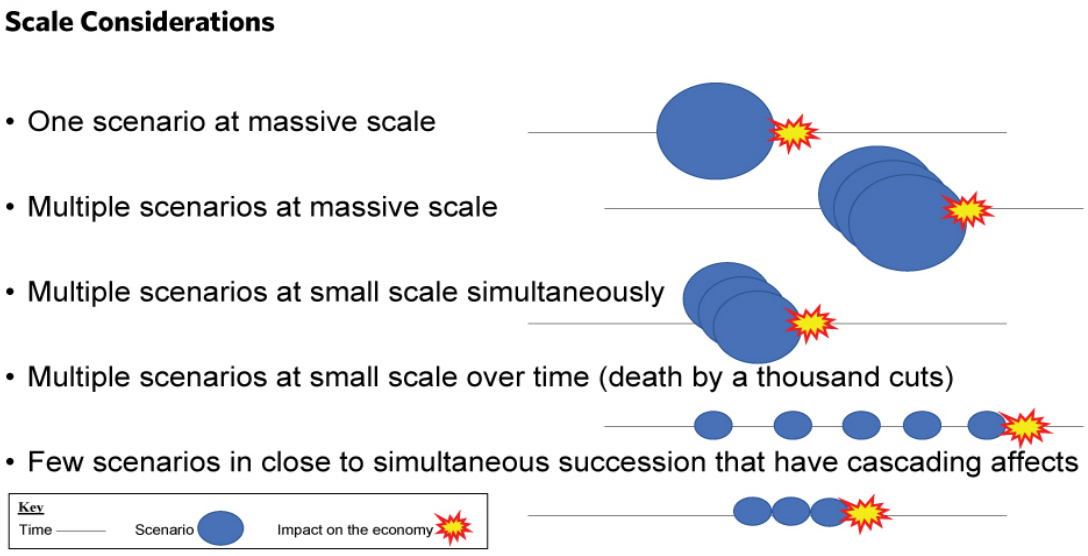
1. ზეგავლენას უნდა ახდენდეს რამდენიმე მსხვილი ზომის ორგანიზაციაზე ერთდროულად, ან მრავალ მცირე და საშუალო ზომის კომპანიაზე;
2. მოვლენის მასშტაბი და ზეგავლენა როგორც კომპანიაზე, ასევე, პოტენციურად ფართო საზოგადოებაზე, უნდა იყოს მაღალი.

სხვა გასათვალისწინებელ საკითხებში უნდა აღვნიშნოთ ისიც, რომ სისტემური კიბერრისკი სხვადასხვა ფორმით შეიძლება წარმოიშვას. სისტემაში შეიძლება განვითარდეს როგორც ერთი მსხვილი მოვლენა ან სცენარი, რომლის შედეგად ეკონომიკის ერთი ან რამდენიმე სექტორი შეიძლება თითქმის ერთდროულად შეფერხდეს ან შეჩერდეს. იგივე ეხება სახელმწიფო სექტორსაც, როდესაც საქართველოში 2008 წლის აგვისტოს ომის დროს, სახელმწიფოს პრაქტიკულად ყველა განშტოება მოექცა კიბერშეტევის ქვეშ. ანალოგიურად, იშვიათად დასაშვებია, მაგრამ პოტენციურად შესაძლებელი, რამდენიმე მასშტაბური მოვლენის ერთდროულად განხორციელება, რამაც შეიძლება დიდი, ან ტოტალური ზეგავლენა იქონიოს ქვეყანაზე. დიაგრამა N2-ში ასახულია ასეთი სცენარები.

ამავდროულად, სისტემური კიბერრისკი პოტენციურად შეიძლება განვითარდეს მაშინაც, როდესაც მრავალი შედარებით მცირე ზომის მოვლენა ერთდროულად ხდება ქვეყანაში, ან ეკონომიკის რომელიმე სექტორში. გასათვალისწინებელია, რომ გარდა ამისა, სისტემური რისკის წარმოშობა შეიძლება დაუკავშირდეს დროში გაწელილ სცენარებსაც, რომელსაც მთლიანობაში საგრძნობელი უარყოფითი ზეგავლენა შეიძლება ჰქონდეს ქვეყნის სხვადასხვა სექტორზე.

დიაგრამა 2. კიბერრისკის მასშტაბთან დაკავშირებული საკითხები

წყარო: კარნეგის საერთაშორისო მშვიდობის ცენტრი

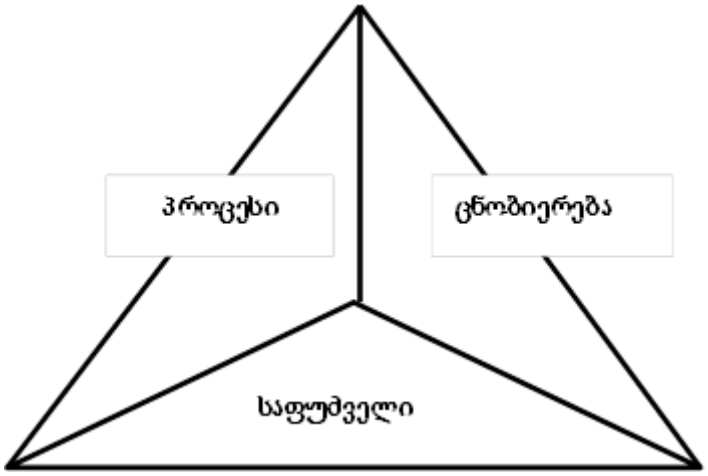


როგორ უნდა შემცირდეს სისტემური კიბერრისკი?

იმისათვის, რომ სისტემური კიბერრისკი შემცირდეს, საწყის ეტაპზე მნიშვნელოვანია ორგანიზაციებში საოპერაციო და ტექნოლოგიური რისკის მართვის ჩარჩოს დანერგვა/განვითარება. კიბერრისკის შემცირება ვერ მოხერხდება, თუ თითოეული კომპანია რისკების მართვის სათანადო ჩარცოს არ დანერგავს. ეს პროცესი შეგვიძლია წარმოვიდგინოთ როგორც სამ-ფერდიანი პირამიდა რომელიც დიაგრამა N3-შია ასახული.

დიაგრამა 3: საოპერაციო და ტექნოლოგიური რისკების მართვის სამი ძირითადი პრინციპი

წყარო: ვესტერმანი და ჰანტერი



საოპერაციო და საინფორმაციო ტექნოლოგიების რისკების(მ.შ. კიბერრისკის) მართვის პროგრამა აგებულია სამ (3) ძირითად პრინციპზე. ზემოაღნიშნული სამი პრინციპი ერთმანეთთან არის დაკავშირებული და შეერთებული იმისათვის, რომ გაუმჯობესდეს ორგანიზაციის რისკის პროფილი და მოცემული რისკის პროფილი კონტროლის ქვეშ მოექცეს. საოპერაციო რისკის მართვის სამი ძირითადი პრინციპია:

1. **საფუძველი** - დანერგილი ტექნოლოგიური ფუძე/ფუნდამენტი ინფრასტრუქტურისა და პროგრამული ტექნოლოგიების და ასევე მხარდამჭერი პერსონალის და პროცედურების, რომელიც კარგად არის გათვითცნობიერებული, კარგად მართული და სირთულის თვალსაზრისით, მაქსიმალურად მარტივი და არა-კომპლექსური. მისი სირთულე არის პროპორციული ორგანიზაციის საქმიანობასთან.

2. **კარგად დაგეგმილი და განხორციელებული რისკების მართვის პროცესი** რომელიც ორგანიზაციას უზრუნველყოს ერთიან ხედვას ყველა რისკებზე, რაც კომპანიის ხელმძღვანელობას შესაძლებლობას აძლევს, რომ პრიორიტეტულობის გათვალისწინებით განახორციელოს სათანადო ინვესტიციები რისკების მართვის მიმართულებით, მაგრამ ამავდროულად აძლევდეს ქვედა დონის მენეჯმენტს დამოუკიდებლად რისკების მართვის უნარს თავიანთი შესაბამისი მიმართულებებით.

3. **რისკების მიმართ ინფორმირებული და ცნობიერება-ამაღლებული საორგანიზაციო კულტურა**, რომელშიც ყველას გააჩნია სათანადო ცოდნა და რისკის აღქმა, სადაც ღია და მუქარებისგან თავისუფალი დისკუსიები წარმოადგენს ნორმას. ე.წ. ბრეინსტორმინგი. ეს

კომპანიებს იმის შესაძლებლობას აძლევს, რომ კომპანიამ ობიექტურად განიხილოს ყველა პოტენციური და რეალური რისკი, რის წინაშეც შეიძლება იდგას ორგანიზაცია.

გარდა მოცემული სამ-საფეხურიანი მართვის პირამიდისა, გასათვალისწინებელია როგორც სახელმწიფოს, ასევე ეკონომიკის თითოეული სექტორის მარეგულირებლის როლიც. ქვეყანაში აუცილებელია იყოს განსაზღვრული კრიტიკული ინფრასტრუქტურა, რომელიც მოიცავს როგორც სახელმწიფო, ასევე კერძო სექტორებს. იმისათვის, რომ კერძო სექტორში არსებული კომპანიები სათანადადოდ იყვნენ გათვალისწინებული კრიტიკულ ინფრასტრუქტურაში, საჭიროა კრიტიკული სექტორების განსაზღვრაც. ასეთ სექტორებს შეიძლება წარმოადგენდეს ენერგეტიკის სფერო, წყალმომარაგება, ფინანსური სისტემა და ტრანსპორტი. სასურველია, რომ ეკონომიკის თითოეულ სექტორს გააჩნდეს შესაბამისი საზედამხედველო უწყება, რომელიც უზრუნველყოფს კიბერრისკის საკითხის კოორდინირებას, ან საჭიროების შემთხვევაში, ზედამხველობას.

უნდა აღინიშნოს ისიც, რომ სისტემური კიბერრისკის მართვის პროცესი ვერ იქნება სრულყოფილი თუ ქვეყანაში, ისევე როგორც თითოეულ კრიტიკულ სექტორში არ იარსებებს კიბერ-ინციდენტებთან დაკავშირებული ინფორმაციის დროული გაცვლის მექანიზმი. ზუსტად ასეთ მექანიზმს არაერთ ქვეყანაში წარმოადგენს ინფორმაციის გაზიარებისა და ანალიზის ცენტრები(ე.წ. ISAC). ამ ცენტრების უპირატესობა ის არის, რომ მოცემული ორგანიზაციები ერთის მხრივ აერთიანებენ კერძო სექტორში შემავალ არაერთ კომპანიას, უზრუნველყოფენ ინფორმაციის სწრაფ გაზიარებას წევრ კომპანიებს შორის და მეორეს მხრივ კოორდინირებას უწევენ კიბერინციდენტებთან დაკავშირებულ პროცესებს.

სისტემური კიბერრისკის სამართავად, საჭიროა როგორც კომპანიების ჩართულობა რისკების მართვის პროცესში, ასევე სახელმწიფოს მხრიდან კოორდინირებული მოქმედება კერძო სექტორთან ერთად. იმ ქვეყნებში, სადაც კერძო-საჯარო ურთიერთობა სახელმწიფოსა და კერძო სექტორს შორის კარგად არის ჩამოყალიბებული და განვითარებული, კიბერრისკი უფრო ეფექტურად იმართება. ეს მოიცავს სისტემურ კიბერრისკსაც.

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