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Applications of Stochastic Processes and Mathematical Statistics to Financial Economics and Social Sciences III

Tbilisi Science and Innovation Festival 2018

Conference Materials, 28-29 September, 2018, Tbilisi, GAU

Business School

Business Research Scientific Center

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შემთხვევითი პროცესებისა და მათემატიკური სტატისტიკის გამოყენებანი ფინანსურ ეკონომიკასა და სოციალურ მეცნიერებებში III

Applications of Stochastic Processes and Mathematical Statistics to Financial Economics and Social Sciences III

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კონფერენციის მასალები
Conference Materials

ბიზნესის სკოლა
Business School

ბიზნეს კვლევების სამეცნიერო ცენტრი
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SYMBOLIC IMPRINTING AND TRIGGERS IN ORGANIZATION CONTEXT

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ABSTRACT

The paper discusses the place of symbolic imprinting and triggers in organization, underlining their significant role in symbolic results of managerial actions and decisions. Symbolic imprinting constitutes stamping in memory some transferring values and meaning interpretations, which define the actions and behaviors. Triggers in their turn are the external environment signals which make imprints work out in specific thinking, behavioral, and communication mechanisms.

Key words: organizational symbolism, symbolic imprinting, triggers, symbolic management.

> It is not enough to pick up isolated data of our past experience; we must really *re-collect* them, we must organize and synthesize them, and assemble them into a focus of thought. It is this kind of recollection which gives us the characteristic human shape of memory, and distinguishes it from all the other phenomena in animal or organic life [1]
>
> E. Cassirer

Nowadays organizations represent complex systems with many interrelated elements, and within the system it is included human (employee or manager). Human being is a complex system itself and deploys its elements as well in the synergy of organizational life. This human as a system element is 'homo symbolicus' with eternal need to cognize and understand the surrounding reality. In human perspective the need for world cognition becomes the problem of world construction with the help of symbolic forms. Mostly known

symbolic forms in human cultural expression are science, religion, language, philosophy, history, myth. Human being needs symbolic transformation, so transforming empirical and emotional data into symbols, this is the way of person – environment interaction. Susan Langer in her observation of symbolic underlines that symbols present not the objects but are transferring specific concept on those objects [2]. So, symbols and symbolic in organization transfer the concept and context on the common objects, operations and actions.

Human being surrounds his/her activity by many rituals, when people get together they 'organize' these rituals in such a way that they constitute tradition, and in some sense with the mental field intervention as well culture (something defining behavior opposite to nature). These surroundings have symbolic nature, and brightly are deployed in organizational life, where group of people with specific structure define specific purpose. The 'people' component creates additional complexity for the system managers need to plan, organize, lead and control in.

Where there is a person, there always will be symbolic tension in meanings, even words constitute this symbolic tension, like the energy of meaning or content transfer. Ernst Cassirer defines that symbols cannot be reduced to mere signals. Signals and symbols belong to two different universes of discourse: a signal is a part of the physical world of being; a symbol is a part of the human world of meaning, as signals are 'operators'; symbols are 'designators' [3].

Imprinting is mostly studied in animals, less in human psychology. In its genesis it means 'stamped in' impression, tactile, visual or sound memory. It is like the selective fixed information in memory. All people have periods when their memory is sensitive to imprinting, so some environmental events can be stamped in, and influence decision, behavior and even values choice in the future. Symbolic context of organizational life (interactions, communication, relationships, decisions, job satisfaction etc.) creates the situation for human sensibility, as it is the situation of suspense (border situation). Mircea

Eliade specified that rituals open the suspense situation of a humankind; it is a situation where human realizes the place in the world. Symbols' function in Eliade's interpretation, is to open the whole reality, other means are not able to do [4]. Karen Svasian states that there is no symbol itself in any of its forms; its being is in metamorphose, is forms transform [5]. So, symbolic situation is a situation of transformation, and symbolic imprinting gets its place in these situations in organization. Social imprinting is presented in the number of imprints among people, like basic principles, values, stereotypes, which in their turn define the attitudes towards leadership, discipline, ethical issues etc. Symbolic imprinting means stamping in memory some transferring values and meaning interpretations, which define the actions and behaviors. For example, in organization symbolic imprints can be values important for the group of people and successes they experience together. The symbolic imprints are deployed in meanings and understanding employees hold about organizational goals, priorities, can be transferred into job satisfaction, attitudes and decisions.

Symbols in imprinting process can be understood as meaning triggers. Triggers are the factors of the external environment which start specific behavioral mechanisms. In some sense symbolic imprints are displayed in the reactions on triggers in organization. So, in the specific situation people release these imprints in its understanding and interpretation, which in its way lead to the specific behavior. Marshall Goldsmith [6] in his view on triggers in life and workplace underline the importance of self-awareness in making decisions and choosing the style of behavior. Struggling with triggers is especially difficult what they have symbolic context and are based on the symbolic imprints, not changeable over time.

Managers in organization usually deal with the results of triggers and symbolic imprints within the context of employees' behavior and learning. Additionally, organizational culture creates the symbolic context for the potential suspense situations, where new imprints can originate. As Mats Alvesson in his book organizational culture differentiates organizational results, following Pfeffer's symbolic management, as objective (measurable) and symbolic

(attitudes, emotions, values, perception). Symbolic results from managerial actions and decisions create the opportunity for creating mutual understanding of the organizational goals and social reality events [7]. Symbolic imprinting and triggers play a very important role in this organizational context. Organizational culture is analyzed by Alvesson like the complex model of meanings, ideas and symbols [Ibid.]. Symbol creates a meaning context, transferring context of reality, and symbolic imprints make employees understand specific goals and events on the irrational level. Triggers can make symbolic imprints work out, and symbolic management can deploy the mechanisms of awareness in such a way that they will help employees get better results and stop the to demonstrate blindly some programs in their behavior.

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BEHAVIORAL PERSPECTIVE IN MANAGEMENT

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ABSTRACT

The paper covers some main problematic issues important for management in organizations, specifically within the human behavior perspective. There are discussed the concepts of the behavioral perspective in management, underlining the importance of human element in organization achievement of results and success.
Keywords: management, organizational behavior, attitudes, and values.

Management is the art of getting
things done through people [1]
M.P. Follet

Various definition of management does not run contrary to one another. Management is the sum-total of all activities that determine objectives, plans, secure men, material, machinery cheaply, put all these resources into operations through organization, direct and motivate people at work, supervises and control their performance and provide maximum prosperity and happiness for both employer and employees.
Each of us is a student of behavior. We are aware that a certain type of behavior is linked to certain types of responsibilities. As we mature, we expand our observations to include the behavior of others. We develop generalizations that help to predict and explain what people do and will do. *How accurate are these generalizations?* Some may represent extremely sophisticated appraisals of behavior and prove highly effective in explaining the behavior of others. Most of us also carry several beliefs that frequently fail to explain why people do what they do. As a result, systematic approach to the study of behavior can improve an individual's explanatory and predictive abilities.

The environment of business is always changing. With increases in workload and responsibilities, managers have found that their importance in the work environment has also increased. To help managers and supervisors learn more about the complexity of the new workforce many different areas of study have been developed. One area of study that

has increased in importance over the years is the study of organizational behavior. Organizational behavior tries to examine different types of workers in all types of different situations. There are many reasons for the importance of organizational behavior in an organization. *First*, most people are born and educated in organizations, acquire most of the material possessions from organizations, and die as members of organizations. In addition, we can be consumers, employees, or investors in an organization. *Second,* the study of organizational behavior can greatly clarify the factors that affect how managers manage. *Third*, the value of organizational behavior is that it isolates important aspects of the manager's job and offers specific perspectives on the human side of management: people as organizations, people as resources and people as people. *Finally*, an understanding of organizational behavior can play a vital role in managerial work. Organizational Behavior introduces a comprehensive set of concepts and theories, it has to deal with a lot of commonly accepted 'facts' about human behavior and organizations that have been acquired over the years, like "you can teach an old dog new tricks". "Two heads are better than one". These facts are not necessarily true. Organization Behavior does offer challenges and opportunities for managers since it focuses on ways and means to improve productivity, minimize absenteeism, increase employee job satisfaction, etc. Organization Behavior can offer managers guidance in creating an ethical work climate. This is because organizational behavior can improve prediction of behavior.

The behavioral perspective of management (sometimes called the "humane perspective") is a way to manage a corporation where the employees are viewed as social beings with complex needs and desires as opposed to just units of production. This perspective places an emphasis on the social networks found in a corporation and uses gratification to provide motivation in the workplace. The better our human relations, the more likely we are to grow both professionally and personally. Behavioral management shifted attention from the largely mechanistic view of employees and work held in classical management to the psychology, attitudes and behaviors of individuals. Thus, it is concerned with recognition of the importance of human element in organizations. It revealed the importance of social and psychological factors in determining worker's productivity and satisfaction. Understanding personal needs and motives, how to improve decision-making capabilities, how to respond to and control stress, how to better communicate with others and the way in which career dynamics unfold can all be of enormous benefit to individual managers. Organizational behavior once again provides useful insights into these concepts and processes. Attitude is the major factor, which affect the behavior of a person or an organization. It manipulates the perception of objects and people, exposure to and comprehension of information, choice of friends, co-workers and so on. The importance of attitudes in understanding psychological phenomenon was given formal recognition early in the history of social psychology.

Attitudes may be defined in two ways conceptual and operational. Even there is a quite difference in the conceptual definition of the term attitude. The term attitude first entered in the files of social phenomenon, it was natural to conceive of attitude as tendency, set, or readiness to respond to some social objects. Some authors define attitude as a mental and

neutral state of readiness, organized through experience, exerting directive or dynamic influence upon the individual's response to all objects and situations with which it is related. Many researchers have defined attitude in terms of effect and evaluation. For example, Krech and Crutchfield define attitude as an enduring organization of motivational, emotional, perceptual, and cognitive processes with respect to some aspect of the individual's world [2].

Thus, attitudes are beliefs imbued with emotional and motivational properties and are expressed in person's favorability towards an object. Evaluation consists of attributing goodness-badness or desirable-undesirable qualities to an object. In addition to conceptual approach, there is operational approach in defining the term attitude. The concept of attitude is viewed in several ways; attitude should be regarded as synonymous with evaluating meaning.

In practice, the term attitude often is used in a generic sense to any reports of what people think or feel or the ways in which they intend to act, thus our personalities and values play a large role in our attitudes as well. There are so many theories that have been projected to explain the attitude formation and change. Although, these theories have many limitations, they provide useful thinking about the processes underlying attitude formation. These components are organized into major groupings according to the nature of the psychological processes postulated to underlying formation and change of attitudes. We all hold definite views about things and people - feelings referrers to as attitudes. Formally, we define as attitude as a relatively stable cluster of feelings, beliefs, and behavioral predisposition toward some specific target. Attitude consist of three major components: an evaluative component, a cognitive component, behavioral component.

The evaluative component: the most obvious component of attitudes is how we feel about something. This aspect of an attitude, its evaluative component, refers to our liking or disliking of any target - be it a person, thing, or event (what might be called the attitude object, the focus of the attitude). You may, for example, feel positively or negatively toward your boss, co-workers, or the company logo. In fact, anything can be an attitude object.

The cognitive component of an attitude involves more than feelings, however, also knowledge - things you know about an attitude object. For example, you might believe that your company just lost an important contract, or that a co-worker doesn't really know what he is doing. These beliefs may be completely accurate or inaccurate, but they still comprise the personal knowledge that contributes to your attitude.

The behavioral component: naturally, what you believe about something and the way you feel about it will influence the way you are predisposed to behave. For example, if you believe that your boss is crook, and you dislike this, you may be inclined to report him to the authorities and to begin looking for a new job, although you may dislike your unethical boss, you might not act against him. Hence, attitude is not perfect prediction of behaviors.

The attitude is learned. Though there are different approaches as how learning works and is acquired by individuals, generally it is held that individuals learn things from environment in which they interact. Thus, for attitude formation, all those factors from which people

learn must be considered. Individual acquire attitudes from several sources, but the point to be stressed is that the attitudes are acquired, but not inherited.

Values play an important role in attitude originating. People develop values from different sources, e.g. parents, friends, teachers, society, and religion. Parents are the initiators of values. Every family has certain values, which are inculcated in the children. Parental guidance paves the value systems on which children develop their attitude and behavior.

Good company helps develop good values. Good values have become strong forces for good behavior and give satisfaction as they help one to perform better.

Employees have certain values in life. They view life from different angles which are reflected in their work performance. Values refer to the basic convictions, which lead to formation of conduct or social preferences. Values are a combination of different attitudes and attributes of individuals. They help employees decide what is right, good, desirable and favorable and so on. One value may be useful for a person but may not be effective for others. The intensity attributes or value system has relative importance. Values are code of conduct developed by an individual and the social system. Honesty, self-respect, equality, sincerity, obedience, truthfulness, etc. are various examples of values. Values are significant in organizational behavior. Employees behave properly if they give importance to values. Employees would be aware of what ought to be done or what ought not to be done by giving importance to values. Behavior is modified only if people are aware of right or wrong things, which are the deciding components of values. Values influence objectives too, because values shape the attitude and behavior of employees. For example, obedient employees perform efficiently without creating any problems for the management. Disciplined employees feel directed toward objectives.

Donald D. White and David A. Bedner have defined value as a "concept of desirable, an internalized criterion or standard of evaluation a person possesses. Such concepts and standards are relatively few and determine or guide an individual's evaluations of the many objects encountered in everyday life" [3]. Values are important. Individuals learn values as they grow and mature. They may change over the life span of an individual develops a sense of self. Culture, societies and organization shape values. Organization recruiting job candidate should pay careful attention to an individual's values.

According to Rokeach, [4] values represent basic conviction that a specific mode of conduct or end state of existence is personally or socially preferable to an opposite or converse mode of conduct or end state of existence. Values are learnt from the society and hence are acceptable to the society as preferred 'modes of conduct' or 'end states'. Values are stable and long-lasting beliefs about what is important in a variety situation. Values influence our priorities, preferences and our actions. We all have a set of values that form a value system. This system is identified by the relative importance assigned to such values as pleasure, self-respect, honesty, freedom, equality. Consequently, values of an individual influence attitudes and behavior.

In the organizational context, values are important because they influence employee conduct and behavior. They play a vital role in behavioral disposition and the decision-making

behavior of an individual. Values influence behavior in several ways. If an employee feels that payment of wages based on performance is right, s/he cannot accept payment of wages based on seniority, s/he becomes indifferent to such a system followed in the organization. Therefore, the values of individuals need to be studied, so that employees' values could be matched with that of the organization. The values at work place may be defined as the perception of what is preferable from among the alternative modes of conduct or end states with respect to one's work. Managers must study values because they are the foundations for understanding a person's attitudes, perceptions, motivation and behavior in the organization. Values and attitudes are not separated by size or space. Instead values and attitudes are created and lived through the interaction of the community, organization, business unit, and the individual. If one part is not living by the values created by the organization or the organization's values do not fit the overall culture, the organization will not be as successful as it should be. Positive values and attitudes take much time, coordination and effort. It takes the community to provide the framework. It takes community to provide the framework, the organization to fill the details, the business unit to live those values daily, and the individual to have a positive attitude to use those values to guide in their everyday performance and decision-making.

Managers usually are viewed as standards setters and values disseminators in organizations. It is important for them to understand the role values play in forming attitudes and deploying behaviors of the employees.

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სამსახურით კმაყოფილება/უკმაყოფილების განმსაზღვრელი კრიტერიუმები

(კერძო და საჯარო სტრუქტურა)

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შესავალი

სამსახურით კმაყოფილება დღევანდელ ბიზნეს გარემოში მეტად საჭირბოროტო საკითხია. მსოფლიოში თანამედროვე ორგანიზაციები მივიდნენ იმ აზრამდე, რომ მათი წარმატების უმნიშვნელოვანეს რგოლს მათივე თანამშრომლები წარმოადგენენ. ბოლო პერიოდში ჩატარებული კვლევებიდან ჩანს, რომ რაც უფრო კმაყოფილია თანამშრომელი თავისი სამუშაოთი მით უფრო მეტია მისგან სარგებლის მიღების ალბათობა.

სამსახურით კმაყოფილების თეორიები მჭიდრო კავშირშია მოტივაციის თეორიებთან, როგორიც არის მასლოუს საჭიროებების პირამიდა, ჰერზბერგის მამოტივირებელი-ჰიგიენური ფაქტორების თეორია, სამუშაოს მახასიათებლების მოდელი და სხვა.

ამერიკის ფსიქოლოგიური ასოციაცია (American Psychological Association), ჰარვარდის უნივერსიტეტის ბიზნეს მიმოხილვის ჟურნალი (Harvard Business Review) და სხვა მრავალი წამყვანი ინტერნეტ თუ ბეჭდური მედიასაშუალებები მუდმივად აქვეყნებენ სტატიებს კომპანიებში თანამშრომლების სამუშაოთი კმაყოფილების შესახებ.

საინტერესოა, აღნიშნული კვლევის ჩატარება საქართველოშიც. კვლევის შედეგებიდან გამომდინარე შესაძლებელი იქნება წარმოჩინდეს მნიშვნელოვანი ინფორმაცია, რაც საშუალებას მისცემს ორგანიზაციებს უკეთ მოახდინონ თანამშრომლების მოტივაცია და პროდუქტიულობის გაზრდა, ხოლო დასაქმებულთათვის სამუშაო გახდეს მეტად საინტერესო და სასიამოვნო.

მნიშვნელოვანი სიტყვები: კერძო, საჯარო, მოტივაცია, კმაყოფილება, მოტივაცია

კვლევის მიზანი

კვლევის მიზანია გამოარკვიოს და დაადგინოს კერძო და საჯარო სექტორებში თანამშრომლების კმაყოფილება/უკმაყოფილების კრიტერიუმები.

კერძო და საჯარო სექტორი ერთმანეთისგან მნიშვნელოვნად განსხვავდებიან. განსხვავებულია მათი სამუშაო მიდგომები, მოტივაციის სისტემები, კორპორატიული კულტურა, თანამშრომლების მიმართ დამოკიდებულება და პირიქით, თანამშრომლების დამოკიდებულება ორგანიზაციის მიმართ. აღნიშნული გახდა იმის წინაპირობა, რომ კვლევაში ეს ორი სექტორი ერთმანეთისგან გაგვემიჯნა და გამოგვეკვლია თანამშრომლების აზრი ორივე მიმართულებაში.

კერძო და საჯარო სექტორებში ჩატარებული კვლევა, საშუალებას მოგვცემს პირველ რიგში ერთმანეთ შევადაროთ შედეგები და ვნახოთ სად არის განსხვავებები. აღნიშნული ფაქტი,

საშუალებას მოგვცემს ზუსტად ჩამოვაყალიბოთ რეკომენდაციები ორივე მიმართულებისთვის.
კვლევის შედეგებიდან გამომდინარე, შესაძლებელი იქნება დავგეგმოთ მომავალი კვლევების აუცილებლობა.
კვლევის შედეგად შეგვეძლება შევიმუშაოთ პრაქტიკულად გამოსაყენებელი რეკომენდაციები როგორც ორგანიზაციებისთვის, ისე მათი თანამშრომლებისთვის რაც ხელს შეუწყობს მათ მომავალ კეთილდღეობას.
კვლევა ასევე საინტერესო იქნება მეცნიერული განხილვა/ანალიზის მიზნით. მისი შედეგების გამოყენება შესაძლებელი იქნება სხვადასხვა მიმოხილვით სტატიებსა თუ კვლევებში.

კვლევის ამოცანები

კვლევის ამოცანაა, გამოვარკვიოთ არიან თუ არა კმაყოფილი ადამიანები საქართველოში თავიანთი სამუშაოთი და თუ არ არიან რა არის ამის ხელის შემშლელი ფაქტორები. აღნიშნულიდან გამომდინარე კვლევის ამოცანებია:

- კმაყოფილების განმსაზღვრელი კრიტერიუმები კერძო და საჯარო სექტორში
- უკმაყოფილების განმსაზღვრელი კრიტერიუმები კერძო და საჯარო სექტორში
- პროფესიით მუშაობის პრეცედენტები და მოტივატორები/რეჯექტორები
- კერძო და საჯარო სექტორში დასაქმებული ადამიანების კმაყოფილება/უკმაყოფილების მიზეზებს შორის განსხვავებები.

ჰიპოთეზა

დღესდღეობით საქართველოში არსებული არც თუ სახარბიელო ეკონომიკური მდგომარეობიდან გამომდინარე, არსებობს უმუშევრობის პრობლემა. ოჯახების უმრავლესობა ცდილობს, რომ იქონიონ საარსებო წყარო.
ასეთ გარემოში ხშირად არის სიტუაცია როდესაც ადამიანები იწყებენ სამსახურს არა საკუთარი პროფესიით და არა საკუთარი ინტერესების და სიამოვნების შესაბამისად. ამ შემთხვევაში, პირად სურვილებს სძლევს საარსებო შემოსავლის არსებობის საჭიროება, როგორც ეს მასლოუს საჭიროებების იერარქიაშია მოცემული.
კვლევის ჰიპოთეზაა, რომ საქართველოში მომუშავეთა დიდი ნაწილი ორივე სექტორში, ნაკლებ ყურადღებას აქცევს სამსახურით კმაყოფილებას, საყვარელი პროფესიით მუშაობას და თანხმდებიან ნებისმიერ სამუშაოზე რომელიც მათ ფინანსურ საჭიროებებს მოაგვარებს.
აღნიშნული ფაქტი, მიუთითებს ქვეყნის სუსტ ეკონომიკაზე. ორგანიზაციებში, აღნიშნული სიტუაცია ამცირებს განვითარების, კრეატივის, ცვლილებების, ახალი იდეების და რისკიანობის ხარისხს.

სამიზნე ჯგუფი

კვლევის სამიზნე ჯგუფი არის კერძო და საჯარო სტრუქტურში დასაქმებული ადამიანები. აღნიშნული სტრუქტურებიდან გამოკითხული იქნება 26 ადამიანი.
13 საჯარო სექტორის დასაქმებული - აქედან 10 რიგითი თანამშრომელი ხოლო 3 მენეჯერი.
13 კერძო სექტორის დასაქმებული - აქედან 10 რიგითი თანამშრომელი ხოლო 3 მენეჯერი.

კვლევის დიზაინი

თვისებრივი კვლევის ჩასატარებლად არჩეული მაქვს სიღრმისეული ინტერვიუს მეთოდი. სიღრმისეული ინტერვიუ წარიმართება ნახევრადსტრუქტურირებული კითხვების მეშვეობით.
ინტერვიუს წამოწყება მოხდება გეგმის მიხედვით, ხოლო განვითარება შესაძლებელია მოხდეს საუბრიდან გამომდინარე რადგან შესაძლებელია ინტერვიუს დროს მოხდეს ისეთი საკითხების წამოჭრა, რომელიც არ არის გეგმაში შეტანილი თუმცა დაეხმარება კვლევას.

მიუხედავად ამისა, საჭიროა რომ არ მოხდეს საკვლევი თემიდან მნიშვნელოვანი გადახვევა და ინტერვიუერის მიერ უნდა მოხდეს გაკონტროლება.
სიღრმისეული ინტერვიუს მეშვეობით მოვახდენთ მაქსიმალურად ამომწურავი ინფორმაციის შეგროვებას. საინტერესეო იქნება რესპოდენტების შეხედულებების, აღქმების, ღირებულებების და პრეფერენციების შეფასება. ამ მეთოდით ჩატარებულ კვლევაში, შესაძლებელია აღმოვაჩინოთ ისეთი გარემოებები, რომელიც მკვლევარის მიერ თავიდან არ იყო ნავარაუდევი. მსგავსი კვლევების მრავალფეროვნება მოგვცემს იმის საშუალებას, რომ უკეთ შევისწავლოთ ქართული ორგანიზაციები, მათი მუშაობის სისტემა, თანამშრომლების გადმოსახედიდან დანახული დადებითი და უარყოფითი მხარეები და მიღებული ინფორმაცია გამოვიყენოთ როგორ უკუკავშირი მომავალში კიდევ უფრო სწორი და სასარგებლო პრაქტიკის დანერგვის მიზნით.
თუმცა, კვლევის შედეგად მიღებული შედეგები არ არის შესაძლებელი რომ განვაზოგადოთ ფართო გარემოზე. ამისათვის, საჭირო იქნება საკვლევი საკითხის მომავალში რაოდენობრივი კვლევით გამყარება.

კვლევის შედეგების ანალიზი

კვლევის ფარგლებში სიღრმისეული ინტერვიუ ჩაუტარდა ოცდაექვს ადამიანს. ცამეტ კერძო და ცამეტ საჯარო სექტორში მომუშავეს.
კვლევის შედეგები გაანალიზებული იქნა კვლევის ამოცანებთან მიმართებაში.
პირველი ამოცანა იყო კმაყოფილების განმსაზღვრელი კრიტერიუმები კერძო და საჯარო სექტორში. კერძო სექტორში გამოკითხულ თანამშრომლებში გამოიკვეთა ტენდენცია, რომ ადამიანები თავიანთ სამუშაოს აფასებენ საინტერესო და საჭირო საქმედ. მოსწონთ დაძაბული რეჟიმი და ყოველდღიურად განსხვავებული საქმეების კეთება. ძირითადი ნაწილი თვლის, რომ საშუალო ან ოდნავ მეტ ენერგიას ხარჯავს სამუშაოს შესრულებისთვის და აქვთ შინაგანი რესურსი, რომ შეასრულონ კიდევ უფრო მეტი საქმე, თუმცა ამ შემთხვევაში მოტივაცია და მეტი საქმის შესრულების სურვილი უმეტეს შემთხვევაში გაიზრდება თუ მოხდება პოზიციის შეცვლა (თუნდაც იმავე სტრუქტურაში) და დაწინაურება. მუშაობაში ხელისშემწყობ ფაქტორებად სახელდება უპირველესყოვლისა სასიამოვნო კოლექტივი და გარემო რაც გვაძლევს იმის თქმის საშუალებას, რომ დიდი ყურადღება ექცევა ადამიანურ ფაქტორებს. აღნიშნული ფაქტორები გამოიკვეთა ასევე სამუშაო გარემოს აღწერისას, რაც აბალანსებს ისეთ დისკომფორტს როგორიც არის შენობისთვის რემონტის ჩატარების აუცილებლობა და ფიზიკური დისკომფორტი. მუშაობის ხელისშემწყობ ფატორებში აგრეთვე გამოიკვეთა მენეჯმენტის კომპეტენტურობა და კომპანიის მოქნილობა. ასევე გამოჩნდა, რომ ადამიანები კმაყოფილებას განიცდიან სამუშაო რიტმის, პრობლემური საკითხების მოგვარების, გუნდის მართვის და ჩადებული ენერგიიდან მიღებული შექების ან კლიენტის კმაყოფილების შედეგად. აღსანიშნავია, რომ კამყოფილების კრიტერიუმად შემოსავლის განსაზღვრა გამოიკვეთა გამოკითხულთაგან მხოლოდ ერთი პირის ინტერვიუში.
საჯარო სექტორში, ასევე საკმაოდ დიდი ტენდეციაა სამუშაოს დადებითად შეფასების. გამოკითხულთაგან ყველამ დააფიქსირა, რომ მათი სამუშაო არ არის რთული და არ ჭირდებათ დიდი ენერგია მის შესასრულებლად, რადგან საქმე უმეტესწილად არის რუტინული. ხელისშემწყობ ფაქტორებში აქაც აღინიშნება დადებითი გარემოცვა და ადამიანებთან ურთიერთობა, რაც აგრეთვე დაკავშირებულია კმაყოფილებასთან. საინტერესოა სასიამოვნო პროცესის აღწერის პუნქტი, აქ გამოიკვეთა ტენდენცია, რომ

თანამშრომლებს სიამოვნებთ როდესაც აქვთ მეტი პასუხისმგებლობა და თავისუფლება, თუმცა სხვა კითხვებზე პასუხიდან გამომდინარე შეგვიძლია ვივარაუდოთ, რომ ეს არის უფრო მეტად მათი სურვილი რადგან სამუშაო საჯარო სამსახურში არის მეტად რუტინული. ანუ, როდესაც მათ თავიანთი რუტინიდან გადახვევის და უფრო საინტერესო საქმის შესრულება უწევთ, ეს იწვევს ამ პროცესით მათ კმაყოფილებას. ანაზღაურების კმაყოფილებასთან გაიგივება ამ სექტორში არ მომხდარა, შეგვიძლია ვივარაუდოთ რომ ეს არ არის მათი კმაყოფილების კრიტერიუმი (მაგ. იმიტომ, რომ ანაზღაურება არ არის მაღალი) თუმცა ანაზღაურების სქემასთან დაკავშირებულ შეკითხვაში უმეტესობამ აღნიშნა, რომ ხელფასი არის დაბალი მაგრამ მათთვის გასაგები და მისაღებია მიზეზები.

მეორე ამოცანა არის უკმაყოფილების განმსაზღველი კრიტერიუმები კერძო და საჯარო სექტორში. კერძო სექტორში ხელისშემშლელ ფაქტორებად უმეტესწილად დაფიქსირდა ნაკლები დელეგირებები და ბიუროკრატია, აგრეთვე თანამშრომლების დაბალი კვალიფიკაცია. სამუშაო გარემოში უარყოფითად მოქმედებს ქაოსური გარემო და ხმაური, ასევე ერთდოულად რამდენიმე საქმის შესრულების აუცილებლობა, რაც შეიძლება დავაკავშიროთ კერძო კომპანიების ტენდენციაზე მინიმალური კადრების მეშვეობით შეასრულონ მაქსიმალური სამუშაო. ასევე უკმაყოფილებად შეიძლება ჩავთვალოთ, რომ ფიქსირდება მეტის გაკეთების სურვილი თუმცა არა იგივე პოზიციაზე. აქედან ჩანს, რომ თანამშრომლების დაწინაურების სისტემა არ არის მოქნილი.

საჯარო სექტორში უკმაყოფილება არის ორგანიზაციის უმოქმედობაში თანამშრომლების განვითარებაზე ზრუნვის კუთხით. ასევე, ბენეფიტებთან დაკავშირებულ კითხვაში დაფიქსირდა, რომ ფაქტიურად არავითარ დამატებით სარგებელს არ სთავაზობს ორგანიზაცია თანამშრომლებს, მხოლოდ უმნიშვნელო დაზღვევა და სატელეფონო ლიმიტი (ისიც, მხოლოდ ორ შემთხვევაში).

მესამე ამოცანა შეეხება პროფესიით მუშაობის პრეცედენტებს და მოტივატორებს/რეჯექტორებს. კერძო სექტორში ჩანს, რომ საკმაოდ ხშირად მუშაობენ განხვავებული პროფესიის ადამიანები. მათი ნაწილი აღნიშნავს, რომ მიღებული განათლება საერთოდ არ ეხმარებათ სამუშაოს შესრულებაში და ძირითადად თავიანთ გამოცდილებას ეყრდნობიან ხოლო ნაწილი თვლის, რომ მიღებული განათლების კონკრეტულ მიმართულებებს იყენებენ მუშაობის პროცესში. პირდაპირი სპეციალობით მომუშავე გამოკითხულთაგან დაფიქსირდა მხოლოდ ერთი ადამიანი. ამ ფაქტით შეგვიძლია ვივარაუდოთ, რომ კერძო სექტორში საკმაოდ მიღებული პრაქტიკაა სხვადასხვა პროფესიის ადამიანების დასაქმება და სამუშაოს ადგილზე შესწავლა. ეს ფაქტი შეგვიძლია დავაკავშიროთ უკმაყოფილების იმ კრიტერიუმთან სადაც დაფიქსირდა პრობლემა არაკომპეტენტურ კადრებთან დაკავშირებით. გამოკითხულთაგან უკლებლივ ყველა ხედავს საჭიროებას რომ მიიღოს დამატებითი ცოდნა როგორც არსებული სამუშაოს შესახებ, ასევე სხვა სფეროებში. კარიერის გაგრძელების სურვილი ისევე მრავალფერფეროვანია როგორც თანამშრომლების პროფესიები, რაც აგრეთვე შეგვიძლია ამ ფაქტს დავუკავშიროთ, თუმცა ორგანიზაციების მიმართ დამოკიდებულება საკმაოდ ლოიალურია, რადგან სამუშაოს ამ მომენტისთვის აქტიური ძებნა არ დაფიქსირდა. ეს გარემოება, ასევე შეიძლება დაკავშირებული იყოს ბაზარზე სამსახურების მწირ არჩევანთან და ლოიალურობას შესაძლებელია ეს ფაქტიც განსაზღვრავდეს.

საჯარო სექტორში, სპეციალობით მუშაობის უფრო მაღალი მაჩვენებელია. ეს ფაქტი მეტყველებს იმაზე, რომ სახელმწიფო ორგანიზაციები უფრო მეტად აკონტროლებენ სპეციფიური ცოდნის მქონე ადამიანების დაქირავებას. აქვე აღსანიშნავია ერთი გარემოება,

კვლევაში გამოირჩა ერთი გამოკითხული რომლის სპეციალობა არ იყო კავშირში მის სამუშაოსთან და მისი კმაყოფილება სამსახურით იყო გამორჩეულად დაბალი სხვებთან შედარებით თითქმის ყველა ასპექტში. ეს მეტყველებს იმაზე, რომ სახელმწიფო სამსახურში არაპროფესიით მუშაობა იწვევს ადამიანის დემოტივაციას. პროფესიით მომუშავე პირები საკმაოდ ლოიალურობას გამოხატავენ თავიანთი საქმის და ორგანიზაციის მიმართ. ამ სექტორშიც უკლებლივ ყველა გამოკითხულმა დააფიქსირა მზაობა განათლების გაღრმავების კუთხით, უფრო მეტად საკუთარ დარგში. კარიერის გაგრძელების მხრივ ამ სექტორში აზრები უფრო მეტად ბუნდოვანია და უმეტესად მოიაზრება იგივე ორგანიზაცია.

კვლევის ბოლო მიზანი არის, კერძო და საჯარო სექტორში დასაქმებული ადამიანების კმაყოფილება/უკმაყოფილების მიზეზებს შორის განსხვავებების ანალიზი.

საინტერესო იქნება ჯერ მსგავსებებზე გავამახვილოთ ყურადღება. დიდი სხვაობა არ არის ორგანიზაციების მხრიდან მათი თანამშრომლების კმაყოფილებაზე ზრუნვაში. არ გამოიკვეთა ეს ფაქტი არცერთ სექტორში და ორივეგან ფიქსირდება მოთხოვნა მეტი განვითარების საშუალებების არსებობაში (ტრეინინგები, წინსვლა). კომპენსაციის პაკეტები არცერთგან არ არის რადიკალურად მიუღებელი, რათქმაუნდა არის სურვილი, რომ იყოს მეტი ანაზღაურება თუმცა რიგ შემთხვევებში თანამშრომლები თვლიან, რომ მათი შრომა შესაბამისია ანაზღაურებასთან. ეს ფაქტი შეგვიძლია განვამტკიცოთ იმით, რომ თანამშრომლები საკმაოდ მოტივირებული არიან შეასრულონ მეტი საქმე თავიანთი ორგანიზაციებისთვის. გამოკითხულთა უმეტესობამ დააფიქსირა, რომ ისინი ისედაც აკეთებენ იმაზე მეტს ვიდრე მოეთხოვებათ. მსგავსია ადამიანურ ფაქტორებზე ყურადღება, სასიამოვნო კოლექტივი და მენეჯმენტი, რაც დამატებით მოტივს იძლევა მუშაობისთვის. ეს ფაქტი შეიძლება ორგანიზაციის დადებით მხარედაც განვიხილოთ, რადგან ჩანს რომ ორგანიზაციული კულტურა არის ჯანსაღი და არა ტოქსიკური.

რაც შეეხება განსხვავებებს, პირველ რიგში ჩანს სხვაობა სამუშაოს რიტმში. კერძო კომპანიებში უფრო მეტად დაძაბული და სწრაფი სამუშაო გარემოა რაც ამოტივირებს თანამშრომელს, მსგავსი შემთხვევა საჯარო სექტორში ნაკლებად გვაქვს. სამუშაოს ხასიათი საჯარო ორგანიზაციებში უფრო მეტად სტრუქტურირებული და რუტინულია და ეს იწვევს მათ სურვილს, რომ ქონდეთ მეტი პასუხისმგებლობები რაც მოახდენდა მათ მოტივაციას. საჯარო სექტორში გამოკითხული პირები მეტი ენთუზიაზმით აღწერდნენ თავიანთ საქმიანობას, რაც შეიძლება დავაკავშიროთ ფაქტთან, რომ აქ უფრო მაღალია პროფესიით დასაქმების ხარისხი. კერძო კომპანიების თანამშრომლები მეტს საუბრობენ რიტმზე და გამოწვევებზე, საჯარო ორგანიზაციების თანამშრომლები კი სამუშაოს ბუნებაზე.

ჩანს, რომ ორივე სექტორში პროფესიით მომუშავე პირი, რომელსაც ასევე ექნებოდა მაღალი რიტმის და პასუხისმგებლიანი სამუშაო იქნებოდა ყველაზე მეტად მოტივირებული თანამშრომელი. თუმცა, აღსანიშნავია, რომ ორგანიზაციებმა მეტი ყურადღება უნდა დაუთმონ თანამშრომლების განვითარებაზე ზრუნვას, რადგან ჩანს რომ ადამიანებს აქვთ სურვილი გაიღრმაონ ცოდნა თუ ექნებათ ამის საშუალება. შესაბამისად, კვალიფიციური თანამშრომელი გაზრდის პროდუქტიულობას და შეამცირებს უკმაყოფილებას როგორც საკუთარ თავში ასევე სხვა თანამშრომლებში.

კვლევის ჰიპოთეზა ნაწილობრივ დადასტურდა, იმ გარემოებების გათვალისწინებით რომ ნამდვილად არიან ისეთი თანამშრომლები, რომლებიც არ მუშაობენ პროფესიით და მათი მოტივაცია შედარებით დაბალია ან საჭიროებენ დამატებით კვალიფიკაციის ამაღლებას. ასევე, ასეთი ადამიანები პრობლემურ გარემოს ქმნიან სხვა თანამშრომლებისთვის, რაც ამცირებს მათ მოტივაციას. ეს ფაქტი შეგვიძლია ავხსნათ ქვეყნის სუსტი ეკონომიკური

გარემოთი. მეორეს მხრივ, გამოიკვეთა ტენდენცია, რომ პროფესიით მომუშავე თანამშრომლებისთვის ანაზღაურება არ წარმოადგენს მთავარ მამოტივირებელ ძალას და მათ უყვართ საკუთარი საქმე, რის ხარჯზეც მეტ ლოიალურობას იჩენენ თავიანთი ორგანიზაციების მიმართ.

გამოყენებული წყაროები:

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Research Methodologyfor Staff Motivation & Values-Based Framework Within Educational Institutions

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ABSTRACT

This paper defines the need and methodology for research necessary to better understand the real and practical aspects of what motivates university-based employees and what are the underlying values of those employees that affect their performance. How those values coincide with the values of the institution are also relevant in the way the performance of the employees enhance the performance of the institution. The final dissertation will result in a relevant (and usable) menu of performance incentives to match with each employee or set of employees that will then also enhance the values and performance of the institution.

Keywords: motivation, personal values, institutional values, key performance indicators, educational institutions, university, job category

Although much literature and research exist on what motivates employees, there is little information specific to institutions of higher learning, especially universities. Further, even less information exists that ties motivation to the key performance indicators of the universities [1]. The goal of this dissertation is to better understand what motivates employees and how their values coincide with those of the university in order to develop a menu of performance incentives for individual employees that also enhance the value(s) of the institution (university). Management (employers) will then have a set (or menu) of performance incentives to match with each employee or set of employees that will then also enhance the values and performance of the institution.

Survey Participants

Ideally, a multi-university sample would be used, including various sizes of universities and types (public and private). However, this is practically very difficult to do. As such, I have

opted to use only employees of the Georgian American University (GAU) where I am President. As such, I will try to encourage as wide a response throughout GAU's staff and lecturers as possible. By keeping the survey respondents anonymous, I hope to minimize any bias in the responses, i.e., "let's keep the boss happy with positive responses".
Respondents will be identified and aggregated only through the following criteria:

- Age (<20, 20-30, 31-40, 41-50, 51-60, >60), specifically to determine if there are any age-related or generational differences;
- Gender (male, female), which should also determine any existing issues of gender inequality; and
- Job category (management, administrative staff, support staff, lecturer).

These groupings should provide the necessary specificity when analyzing the results and developing a usable and practical menu of incentive measures for a wide variety of employees. From my experience, there are sufficient similarities in sizes and types of universities so that the results are reasonably transferable and can be used with a high degree of compatibility.
The key criterion for grouping the survey respondents will be by job category as one would expect those categories to be more similar in both values and motivating factors. Further, this would be the logical place to apply the set of incentives when negotiating with individual employees.

Survey Development

Employees will be given the survey as one document, containing 2 parts, 1) Motivation Section and 2) Values Section.
1) Motivation Section
The survey methodology is based on Nuttin's survey design[2] where employees complete simple open sentences about what they like/love/want or on the flip side, what they don't like/love/want in order to better understand what motivates them.
The genius and complexity of Nuttin's approach is in the analysis of the answers to the survey and will be described in more detail as part of the analysis of the results.
It was important for me that the survey captured many of the concepts I have observed in over 40 years of working in almost every type of organization and more specifically in almost 15 years of running a private university.
Employees of educational institutions, especially universities, exhibit a relatively complex variety of motivating factors. A private university such as GAU is run like a business with a strict profit motive, but still has the research and public interest characteristics. So,whatare the motivating factors for staff in educational institutions (universities)?
Financial – similar to every other organizational staff member. There are numerous studies which define this characteristic, including "how much is enough" and methods, timing and forms of payment. Equal pay for equal work based on gender is also relevant here.
Benefits – both formal and informal - including vacation, holidays, sick leave, maternity/paternity leave, insurance, pension/retirement, sabbaticals, training, access to

courses, discounts for family members, etc. Is a menu approach to benefits better for motivating staff? Similar to the financial characteristic, there is a plethora of research on this subject.

Status & title, including "trappings" – including public notoriety, facilities, office space, equipment, etc.

Organizational & managerial characteristics– being involved in a complex organizational structure.

Leadership role – being and "being considered" as a leader within the organization, with the students, and outside the institution

Societal aspects

- Imparting knowledge and developing minds and character of students – a combination of assuming one has something which is of value to the students and a truly altruistic concept of wanting to see students grow in knowledge and maturity.
- Improving society in general and a sense of "giving back" – where there is a realization that education is a key aspect in the growth and success of society. Also, many involved in education feel a responsibility to give something back to the same society that gave them whatever measure of success they feel.
- Being associated with a younger generation – there is definitely a motivating factor of being associated with a dynamic younger generation.
- Learning from students – with some similarities to being associated with a younger generation, it can be very motivating to actually learn from the students, especially when there are international and adult students' programs.
- Social interaction – this motivation for social interaction can be manifested with other staff and/or with students.

Generally, the motivating factors listed above are considered positive. I have chosen to avoid any factors that could be construed as negative, although they do exist, as they would not clearly be related to enhancing performance of the institutions. However, some negative characteristics are addressed in the values section.

2) Values Section based on a modified Schwartz Values Survey (SVS)[3]

GAU Corporate Values - To better understand GAU's corporate values, a brief history of GAU is necessary. The original concept of GAU was to create a Doctor of Juris Prudence (JD) program in law to fill the gap in Georgia's legal education. In the early 2000's, Georgia still had an 11-year school program. The academic requirement to become a lawyer was a 4-year bachelor program (LLB degree where the legal subjects were combined with all other general education subjects) and further, there was no bar exam necessary to become a practicing attorney. As such, most of the graduates (some only 20 years old) were lacking both the maturity, full legal education and testing to be effective practicing lawyers. This was the scenario that precipitated the original creation of the new university, which was to be named, Caucasus American University.

When this concept and original business plan for the university was introduced to me, I understood that for financial viability, the university also needed to include a business

school. Although Georgia had 2 other private universities focusing on business, I believed that there were also improvements that could be made to the general business education also. As the business plan was reworked, I agreed to serve as the President of the new university. During that time the Rose Revolution happened in Georgia in 2003, so to emphasize the renewed focus on Georgia, we changed the name of the university to Georgian American University (GAU). After an unsuccessful search for an individual investor to provide startup capital for GAU, I also agreed to be the major investor, secured a private loan and then collected other shareholders that provided a broad range of necessary skills.

My primary reason for involvement in the development of GAU was to give back something to a society that was in transition from Soviet to post-Soviet to reformer and performer, and one in which I had grown to love and call home. As such, and to be a part of the success story for Georgia, GAU had to provide the highest quality, demand-driven, practical programs that were useful to the private, public and NGO sectors. Programs had to be based on successful best Western practices. Students and lecturers had to have academic freedom and excellent communications skills. GAU needed to have ethical practices as its core value, to lead by example and make socially responsible decisions. The university had to be run like a business using Western management principles, with an emphasis on meeting the needs of the organizations that would hire the students and graduates. With a loan to pay off and other shareholders expecting a return on their investment, profitability was also a necessary motive. And further, to ensure its continuing academic excellence and essential to post-graduate degrees, research had to be a key part of the academic programs.

The values survey is based onthe following GAU corporate values described within the statements and intends to understand how those values coincide or differ from those of the individual employees. These are critical in developing individual performance incentives that are in sync with the values of both the university and the employee. These include:

- Performance; effective fiscal management; profitability
- Western standards, programs and management principles
- Value to society;corporate social responsibility (doing the right thing)
- Academic quality; demand driven courses and programs; combination of practical and theoretical knowledge; emphasis on job readiness/communications skills – clients are those organizations that hire students/graduates
- Academic freedom; personal student attention
- Ethical behavior, including Code of Ethics and Conduct
- Research which contributes to advancing scientific knowledge and enhances the brand of GAU.

The next phase of this research is the most interesting for me. I will analyze the survey responses of “my” employees and see if they conform to my perceptions which will include statistical analyses and the creation of the menu of individual performance incentives per job category (and any other statistically valid criteria).

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Georgian-American University
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Real options valuation of hedging strategies

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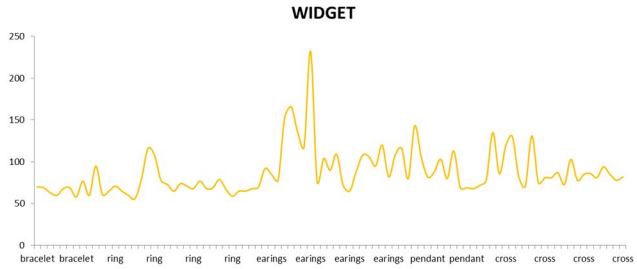
Introduction

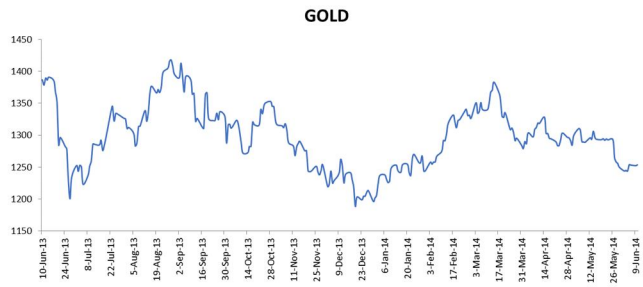
This paper describes valuation of hedging strategies that can be used in managing risks inherent in investment projects. Particularly, protective put is used to hedge against gold price risk inherent in widgets production industry. Because marketed puts on all investments and specific strike prices might not be found, a replicating portfolio principle is used to mimic the payoff from the put with any strike chosen. Cost of rebalancing the portfolio becomes the cost of the put option itself and the total cost of the portfolio is used as the strike price of the real option in the investment project.

This paper follows practical example described in [12] with Georgian producer of gold widgets, Zarapkhana and its gold price risk hedging problem. Opposed to Robust Valuation techniques described in [1] and [12], where project valuation is required under incomplete markets scenario and it is focused on risk mitigation of non-tradable assets such as widgets produced by local manufacturers, this paper focuses on risk management of Gold price risk as an input cost for producing widgets.

Simple Discounted Cash Flow (DCF) Valuation

This section starts with simple discounted cash analysis of gold widgets production in one-period investment horizon. Let's start by looking at past gold prices and widget selling prices by Zarapkhana on per gram basis





Based on sample data of prices, per gram average prices of widget, gold, and resulting mark-up were 87.46, 42.76 and 44.70, respectively. Widgets sale price mark-up of slightly above 50% is caused by factors affecting widget prices other than the gold price. Those factors are largely specific to producer's size, nature of operations, market segments, etc. None of those factors are linked to the risk management problems and, thus, will be ignored in this paper. Consequently, the cash flow from widget production is considered solely as the difference between widget and gold prices. Based on statistical analysis and

Monte Carlo Simulation discussed in [9] next year's expected values and standard deviations of gold and widget prices are given in the table below:

widget	**E[η]**	**$87.66**	*gold*	**E[S]**	**$42.03**
	σ	**54%**		**σ**	**29%**

For simplicity, let discount rate, r, equal to 2% per year. Under one-year period investment horizon present value (PV) of cash flow from widget production is given as:

$$PV = \frac{\eta^1 - S^1}{r} = \frac{87.66 - 42.03}{1 + 0.02} = 44.74$$

In the next sections investment value of the project will be extended to cover the opportunity to hedge the gold price with put option using Real Options Analysis discussed in [2], [3], and [4].

Hedging Strategy Replicating Put Option on Gold Price

Options can be used to guarantee minimum returns from stock or commodity investments such as gold. When you purchase a stock (or gold) and simultaneously purchase a put on the stock (or gold), you are assured that the dollar return from the purchase will never be lower than the exercise price on the put. However, it is not always possible to find marketed puts on all investments and with many different strike prices; in this case the Black-Scholes option-pricing formula is used to replicate a put by a dynamic strategy in which the investment in a risky asset (such as gold) and the investment in riskless bonds changes over time to mimic the returns of a put option.

From Black-Scholes option-pricing formula it follows that a put option on a stock (or gold) is simply a portfolio consisting of a short position in the underlying asset and a long position in the risk-free asset, with both positions being adjusted continuously.

$$\boldsymbol{P = -S_t N(-d_1) + Ke^{-r(T-t)} N(-d_2)}$$

where

$$d_1 = \frac{ln\left(\frac{S_1}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{1-t}} \; and \; d_2 = d_1 - \sigma\sqrt{T-t}$$

Total investment in protective put hedging portfolio then becomes:

$$\boldsymbol{Total\ Wealth = S_t + P_t = S_t - S_t N(-d_1) + Ke^{-r(1-t)} N(-d_2)}$$

$$\boldsymbol{= S_t\left(1 - N(-d_1)\right) + Ke^{-r(1-t)} N(-d_2) = S_t N(d_1) + Ke^{-r(1-t)} N(-d_2)}$$

Finally, proportions invested in the underlying asset and the risk-free bonds can be determined as:

$$\boldsymbol{proportion\ invested\ in\ underlying = \omega_t = \frac{S_t N(d_1)}{S_t N(d_1) + Ke^{-r(1-t)} N(-d_2)}}$$

$$\boldsymbol{proportion\ invested\ in\ risk-free\ asset = 1 - \omega_t = \frac{Ke^{-r(1-t)} N(-d_2)}{S_t N(d_1) + Ke^{-r(1-t)} N(-d_2)}}$$

Continuing example from previous section, protective put hedging strategy can be used to accommodate any strike price for hedging gold price risk in widget production investment project. Then the cost of hedging strategy can be calculated and used as the strike price for valuing investing project as a real option as illustrated in the next section. Applying protective put strategy on the gold price simulated assuming lognormal distribution and using weekly rebalancing of hedging position is summarized in the table below:

		Beginning Week			***Wealth***		
Week	Z	Gold	Put	Omega	Total Wealth	Stock	Bond
0		40.44	3.37	0.50	43.81	21.80	22.01
1	-0.45	39.71	3.65	0.49	43.37	21.11	22.25
2	-0.22	39.36	3.78	0.48	43.13	20.77	22.36
3	0.73	40.51	3.23	0.50	43.74	21.86	21.89
4	0.25	40.91	3.03	0.51	43.93	22.23	21.71
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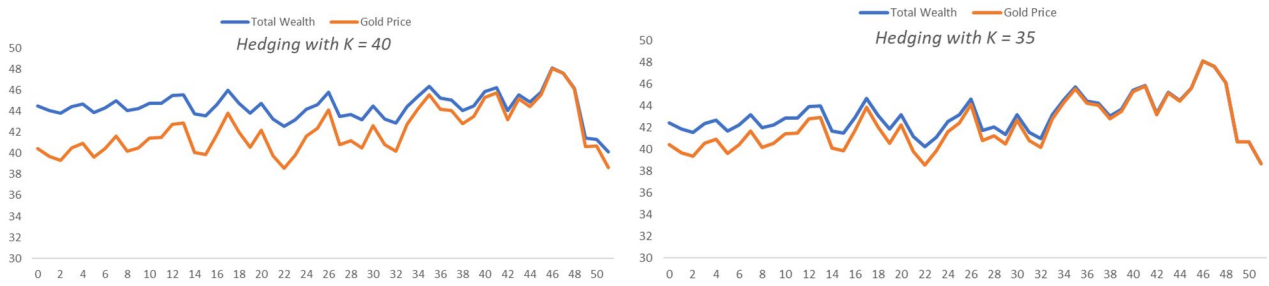
49	-3.07	40.40	0.44	0.56	40.84	22.80	18.04
50	0.02	40.43	0.25	0.57	40.68	23.36	17.33
51	-1.24	38.47	0.90	0.45	39.36	17.84	21.52

Hedging strategy calculations are implemented for different strike prices and the trade-off between hedge value and the cost of hedging must be made by individual investors. Graphical comparison of gold price and hedging portfolio dynamics for two different strike prices, one being close to at the money (K = 40) and one being out of the money (K = 40), is illustrated below:



In the next section, at the money strike of USD 40 will be taken for valuing hedging strategy as a real option in investment project.

Protective Put Hedging Strategy as a Real Option

Following references in [2], [3], and [4], real options valuation analysis is conducted on the investment in widget production project using following parameters: widget price, G = 87.66, lognormal standard deviation, σ = 54%, risk-free rate, r = 0.02, and $\delta\tau$ = 0.08 (4 weeks or 4/52 years) for one-year investment horizon. Opportunity to use protective put hedging portfolio is introduced as a real option of the project and the cost of portfolio construction together with the cost of weekly rebalancing calculated in previous section is taken as the strike price of the option. Resulting binomial tree with widget price evolution and the value of protective put hedge is given below:

	w=0	w=4	w=8	w=12	w=16	…
					159.66	…
				137.43	***111.88***	…
			118.31	***89.85***	118.31	…
		101.84	***71.13***	101.84	***70.75***	…
η	87.66	***55.41***	87.66	***54.88***	87.66	…
H	***42.42***	75.46	***41.75***	75.46	***41.09***	…
		31.14	64.96	***30.34***	64.96	…
			21.90	55.92	***20.98***	…
				14.54	48.13	…
					8.93	…
						…

In the example of Zarapkhana, value of investing in the project using protective put hedging strategy with at the money strike price of USD 40, resulted into value-add of 42.42 per gram. Thus, PV of the project with real option become:

$$\boldsymbol{PV = DCF + Real\ Option\ Value = 44.74 + 42.42 = 87.16}$$

Conclusion

The paper exactly described valuation hedging strategies using real options analysis in investment projects in general, and, particularly, for valuing protective put in the widget production investment project. In the specific example of Zarapkhana producer, addition value from using protective put hedge on gold prices was valued at USD 42.42 per gram, which is almost as much as its original PV obtained by standard DCF analysis.

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Georgian – American University

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Research paper

Portfolio Optimization with Coherent Risk Measures

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Tbilisi

2018

Introduction

The main purpose of the article is to introduce Conditional Value At Risk (CVAR) as a downside risk measure and build a model of portfolio selection process in the mean - CVAR framework for the portfolio returns which are simulated by copula functions. Volatility as a risk measure is ideal in the normal distribution but when dealing with asymmetric distributions, it simply leads to misinterpretation of the risk. It penalizes losses equally to profits of the same magnitude. However, investors are more concerned with a downside risk rather than upside risk which they refer to favorably.

Coherent Risk Measure

According to Artzner et al. a risk measure satisfying the following four axioms is called coherent:

- Subadditivity: if the risk of two portfolios cannot get any bigger than adding two risks separately, then the risk measure is said to be subadditive. In other words, the effect of diversification must be a reduction in risk:

$$\rho(P_1 + P_2) \leq \rho(P_1) + \rho(P_2)$$

- Monotonicity: if the expected returns of one portfolio at any time are greater than the expected return of another, then its risk must be lower.

$$\text{if } P_1 > P_2, \text{ then } \rho(P_1) < \rho(P_2)$$

- Translation Invariance: if A is a deterministic portfolio with a guaranteed return α, then adding A (guaranteed capital) to a portfolio should reduce its risk by the same amount of α.

$$\rho(P + A) = \rho(P) - \alpha$$

- Positive Homogeneity: Scaling the portfolio size by a factor b must simply scale the risk by the same factor.

$$\rho(bP) = b\rho(P)$$

CVAR

Like VAR, CVAR is also a downside risk measure and it measures the average loss beyond VAR with a certain level of confidence. To put it specifically, CVAR is the average return given that the return is smaller than VAR with a certain level of confidence β:

$$CVAR_{\beta} = \frac{1}{1-\beta}\int_{-\infty}^{VAR_{\beta}} f(\omega, r)p(r)dr$$

Where $f(\omega, r)$ is the return function with the distribution of $p(r)$ over a certain time period. VAR is calculated on the same time period. The advantages of using CVAR in portfolio optimization are obvious. It is a coherent risk measure, it is applicable to non-symmetric loss distributions and it is convex and smooth with respect to portfolio positions. In addition, it accounts for risks beyond VAR meaning that it is more conservative than VAR and handles fat tails more effectively.

Optimization

Our goal is to find the weights of the assets in the portfolio which minimizes CVAR on a given portfolio return or maximizes portfolio return on a given CVAR. Let $f(\omega, r)$ be the loss function with the decision vector $\omega \in W \subset R^n$ (portfolio weights) and random vector $r \in R^m$ (returns on portfolio assets). When r has a distribution $p(r)$, then the probability of the loss $f(\omega, r)$ not exceeding a certain threshold level α is given by:

$$\varphi(\omega, \alpha) = \int_{f(\omega,r)\leq \alpha} p(r)dr$$

where φ (cumulative distribution function of loss) is a function of α for fixed ω. φ is nondecreasing with respect to α when $p(r)$ is continuous. So VAR and CVAR can also be defined as follows:

$$VAR_{\beta}(\omega) = \{\alpha \in R: \varphi(\omega, \alpha) \geq \beta\}$$

$$CVAR_{\beta}(\omega) = \frac{1}{1-\beta}\int_{f(\omega,r)\geq VAR_{\beta}(\omega)} f(\omega, r)p(r)dr$$

So CVAR is an integral of losses which are greater than or equal to VAR_{β} divided by $(1-\beta)$ where β is the confidence level. This function is quite difficult to handle since it involves the VAR function. So using this function for optimization process requires the VAR to be calculated first. Rockafellar and Uryasev derived a function which is independent of VAR function, making the optimization process less complicated:

$$F(\omega, \alpha) = \alpha + \frac{1}{(1-\beta)}\int_{r\in R^m}[f(\omega, r) - \alpha]^{+}p(r)dr$$

In order to optimize the function, the integral is approximated by sampling the r returns according to its density. Sampling generates $r = (r_1, r_2, \ldots, r_N)$. We will examine the procedure of sampling returns of dependent returns with copula dependency structures later. Once the returns are generated, in order to optimize the weights and the quantile level α, the function $F(\omega, \alpha)$ should be discretized and approximated by converting it into the following form:

$$F(\omega, \alpha) = \alpha + \frac{1}{1-\beta} \sum_{i=1}^{N} \pi_i [f(\omega, r) - \alpha]^+$$

Where N is the number of simulated scenarios, and π_i is the probability of each scenario. The return of the portfolio is obviously the weighted average of the returns of individual assets in the portfolio. Since the derivation of CVAR is elaborated with the potential loss, we take the negative of the returns:

$$f(\omega, r) = -[\omega_1 r_1 + \omega_2 r_2 + \ldots + \omega_N r_N] = -\omega^T r$$

By assigning equal probabilities to each simulated returns, the optimization problem boils down to:

$$\underset{(\omega,\alpha)}{argmin}\, \alpha + \frac{1}{N(1-\beta)} \sum_{i=1}^{N} [-\omega^T r - \alpha]^+$$

Copula simulation

Copula is a joint distribution function of uniformly distributed random variables. Specifically it is a function of two variables $C(u, v)$ (we only consider two variables for our purposes) for which:

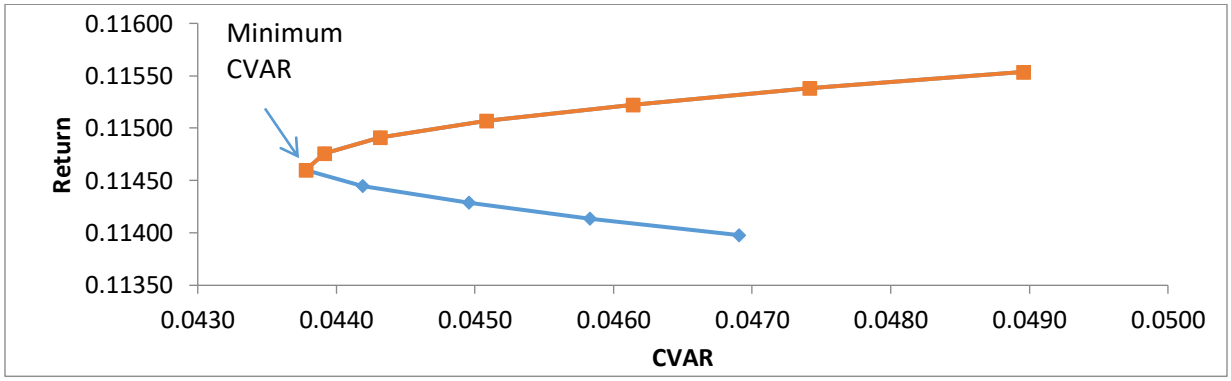
- Domain of C is $[0,1]^2 = [0,1] \times [0,1]$ plane
- $C(0, v) = C(u, 0) = 0$ ও $C(u, 1) = u, C(1, v) = v$ for any u and v. $u, v \in [0,1]$.
- C is a two increasing function, implying that for any $(u_1, v_1) \in [0,1]^2$ ও $(u_2, v_2) \in [0,1]^2$, where $0 \le u_1 \le u_2 \le 1$, $0 \le v_1 \le v_2 \le 1$, the following statement holds:

$$C(u_2, v_2) + C(u_1, v_1) \ge C(u_2, v_1) + C(u_1, v_2)$$

For random variables $X \sim F_X$, $Y \sim G_Y$, let U and V be the uniformly distributed random variables on [0,1] with $U = F(x), V = G(y)$. According to the Sklar's theorem, there exists a function C (which we call copula) for any $x, y \in R$, the following statement is true:

$$H(x, y) = C(F_X(x), G_Y(y))$$

If F and G are strictly increasing marginal distribution functions, then C is unique. There are many patterns of random variables which can be identified by empirical methods and modeled by the copula dependency structure. We use Archimedean copulas to model the dependency and simulate the uniform variables based on the Monte-Carlo method from which we can recover the simulated original variables using the marginal distributions of returns. Once having done so, we obtain the returns vectors as $x_i = F^{-1}(u_i)$, $y_i = G^{-1}(v_i)$ (for two random variables in our case, it can obviously be extended to many). As long as we have the simulated returns, it is possible to calculate the CVAR of a portfolio and minimize the function mentioned in the previous section for every possible return of a portfolio. This gives us the optimal weights based on which we can build the efficient frontier.



Conclusion

CVAR provides a better alternative to variance as a risk measure, especially when returns do not follow the symmetrical distribution. It is a coherent risk measure satisfying very rational requirements unlike variance and VAR. In addition, CVAR has many other advantages over other measures. Simulating returns by copula functions may be crucial if the dependency of the pairs of random variables are perfectly modeled.

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The Itô formula for non-anticipative functionals according to Chitashvili

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Abstract. For non-anticipative functionals, differentiable in Chitashvili's sense, the Itô formula for cadlag semimartingales is proved. Relations between different notions of differentiability of functionals are established.

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1 Introduction

The classical Itô formula shows that for a sufficiently smooth function $(f(t,x), t \geq 0, x \in R)$ the transformed process $f(t, X_t)$ is a semimartingale for any semimartingale X and provides a decomposition of the process $f(t, X_t)$ as a sum of stochastic integral relative to X and a process of finite variation. This formula is applicable to functions of the current value of semimartingales, but in many applications, such as statistics of random processes, stochastic optimal control or mathematical finance, uncertainty affects through the whole history of the process and it is necessary to consider functionals of entire path of a semimartingale.

In 2009 Dupire ([4]) proposed a method to extend the Itô formula for non-anticipative functionals using naturally defined time and space derivatives. The space derivative mesures the sensitivity of a functional $f : D([0,T], R) \to$

R to a variation in the endpoint of a path $\omega \in D([0,T],R)$ and is defined as a limit

$$\partial_\omega f(t,\omega) = \lim_{h\to 0} \frac{f(t,\omega + hI_{[t,T]}) - f(t,\omega)}{h}.$$

Similarly is defined the second order space derivative $\partial_{\omega\omega} f := \partial_\omega(f_\omega)$.

The definition of the time derivative is based on the flat extension of a path ω up to time $t+h$ and is defined as a limit

$$\partial_t f(t,\omega) = \lim_{h\to 0+} \frac{f(t+h,\omega^t) - f(t,\omega)}{h},$$

if this limit exists, where $\omega^t = \omega(.\wedge t)$ is the path of ω stopped at time t.

If a contiuous non-anticipative functional f is from $C^{1,2}$, i.e., if $\partial_t f, \partial_\omega f, \partial_{\omega\omega} f$ exist and are continuous with respect to the metric d (see definition in section 3) and X is a continuous semimartingale, Dupire ([4]) proved that the process $f(t,X)$ is also a semimartingale and

$$f(t,X) = f(0,X) + \int_0^t \partial_t f(s,X)ds + \int_0^t \partial_\omega f(s,X)dX_s + \\ + \frac{1}{2}\int_0^t \partial_{\omega\omega} f(s,X)d\langle X\rangle_s. \qquad (1)$$

For the special case of $f(t,X_t)$ these derivatives coincide with the usual space and time derivatives and the above formula reduces to the standard Itô formula. Further works extending this theory and corresponding references one can see in [3], [8].

Motivated by applications in stochastic optimal control, before Dupire's work, Chitashvili (1983) defined differentiability of non-anticipative functionals in a different way and proved the corresponding Itô formula for continuous semimartingales. His definition is based on "hypothetical" change of variable formula for continuous functions of finite variation.

We formulate Chitashvili's definition of differentiability and present his change of variable formula in a simplified form and for one-dimensional case.

Let $C_{[0,T]}$ be the space of continuous functions on $[0,T]$ equipped with the uniform norm. Let $f(t,\omega)$ be non-anticipative continuous mapping of $C_{[0,T]}$ into $C_{[0,T]}$ and denote by $\mathcal{V}_{[0,T]}$ the space of functions of finite variation on $[0,T]$.

A continuous functional f is differentiable if there exist continuous functionals f^0 and f^1 such that for all $\omega \in C_{[0,T]} \cap \mathcal{V}_{[0,T]}$

$$f(t,\omega) = f(0,\omega) + \int_0^t f^0(s,\omega)ds + \int_0^t f^1(s,\omega)d\omega_s. \tag{2}$$

A functional f is two times differentiable if f^1 is differentiable, i.e., if there exist continuous functionals $f^{0,1}$ and $f^{1,1}$ satisfying

$$f^1(t,\omega) = f^1(0,\omega) + \int_0^t f^{1,0}(s,\omega)ds + \int_0^t f^{1,1}(s,\omega)d\omega_s. \tag{3}$$

for all $\omega \in C_{[0,T]} \cap \mathcal{V}_{[0,T]}$.

Here functionals f^0, f^1 and $f^{1,1}$ play the role of time, space and the second order space derivatives respectively.

It was proved by Chitashvili (1983) that if the functional f is two times differentiable then the process $f(t,X)$ is a semimartingale for any continuous semimartingale X and is represented as

$$f(t,X) = f(0,X) + \int_0^t f^0(s,X)ds + \int_0^t f^1(s,X)dX_s +$$

$$+\frac{1}{2}\int_0^t f^{1,1}(s,X)d\langle X\rangle_s. \tag{4}$$

The idea of the proof of change of variable formula (4) for semimartingalse is to use two times the change of variable formula for functions of finite variations, first for the function f and then for its derivative f^1, before approximating a continuous semimartingale by processes of finite variation.

Our goal is to extend the formula(4) for RCLL (or cadlag in French terminology) semimartingales and to establish how Dupire's and Chitashvili's derivatives are related.

Since the bumped path used in the definition of Dupire's vertical derivative is not continuous even if ω is continuous, to compare derivatives defined by (2) with Dupire's derivatives, one should extend Chitashvili's definition to RCLL processes or to modify Dupire's derivative in such a way that perturbation of continuous paths remain continuous.

The direct extension of Chitashvili's definition of differentiability for RCLL functions is following:

A continuous non-anticipative continuous functional f is differentiable, if there exist continuous non-anticipative functionals f^0 and f^1 (continuous with respect to the metric d) such that $f(\cdot,\omega) \in \mathcal{V}_{[0,T]}$ is of finite variation for all $\omega \in \mathcal{V}_{[0,T]}$ and

$$f(t,\omega) = f(0,\omega) + \int_0^t f^0(s,\omega)ds + \int_0^t f^1(s-,\omega)d\omega_s + \tag{5}$$

$$+ \sum_{s\le t} \big[f(s,\omega) - f(s-,\omega) - f^1(s-,\omega)\Delta\omega_s\big],$$

for all $(t,\omega) \in [0,T] \times \mathcal{V}_{[0,T]}$.

In order to compare Dupire's derivatives with Chitashvili's derivatives, we introduce another type of vertical (or vertical-diagonal) derivative where, unlike to Dupire's derivative $\partial_\omega f$, the path deformation (or perturbation) of continuous paths are also continuous.

We say that a non-anticipative functional $f(t,\omega)$ is differentiable and denote this differential by $D_\omega f(t,\omega)$, if the limit

$$D_\omega F(t,\omega) := \lim_{h\to 0, h>0} \frac{f(t+h,\omega^t + \chi_{t,h}) - f(t+h,\omega^t)}{h}, \tag{6}$$

exists for all $(t,\omega) \in [0,T] \times \mathcal{V}_{[0,T]}$, where

$$\chi_{t,h}(s) = (s-t)1_{(t,t+h]}(s) + h1_{(t+h,T]}(s).$$

Let $f(t,\omega)$ be differentiable in the sense of (5) Then, as proved in Proposition 1,

$$f^0(t,\omega) = \partial_t f(t,\omega) \quad \text{and} \quad f^1(t,\omega) = D_\omega f(t,\omega). \tag{7}$$

for all $(t,\omega) \in [0,T] \times \mathcal{V}_{[0,T]}$.

Thus, f^0 coincides with Dupire's time derivative, but f^1 is equal to $D_\omega f$ which is different from Dupire's vertical derivative in general. The simplest countreexample is $f(t,\omega) = \omega_t - \omega_{t-}$. It is evident that in this case $\partial_\omega f = 1$ and $D_\omega f = 0$. In general, if $g(t,\omega) := f(t-,\omega)$ then $D_\omega g(t,\omega) = D_\omega f(t,\omega)$ and $\partial_\omega g(t,\omega) = 0$ if corresponding derivatives of f exist. However, if $f \in C^{1,1}$ in Dupire's sence, then $D_\omega f$ exists and $D_\omega f = f^1 = \partial_\omega f$.

The paper is organized as follows: In section 2 we extend Citashvili's change of variable formula for RCLL semimartingales and in section 3 we establish relations between different type of derivatives for non-anticipative functionals.

2 The Itô formula

Let $\hat{\Omega} := \mathbb{D}([0,T],\mathbb{R}^d)$ be the set of càdlàg paths, denote by ω the elements of $\hat{\Omega}$, $\omega^t = \omega(\cdot \wedge t)$, B the canonical process, $\mathbb{F}$ the filtration generated by B and $\hat{\Lambda} := [0,T] \times \hat{\Omega}$. We define seminorms on $\hat{\Omega}$ and a pseudometric on $\hat{\Lambda}$ as follows: for any $(t,\omega),(t',\omega') \in \hat{\Lambda}$,

$$\begin{aligned} \|\omega\|_t &:= \sup_{0\le s\le t} |\omega_s|, \\ d_\infty\big((t,\omega),(t',\omega')\big) &:= \big|t-t'\big| + \sup_{0\le s\le T} \big|\omega_{t\wedge s} - \omega'_{t'\wedge s}\big|. \end{aligned} \tag{8}$$

Then $(\hat{\Omega}, \|\cdot\|_T)$ is a Banach space and $(\hat{\Lambda}, d_\infty)$ is a complete pseudometric space. Let $\mathcal{V} = \mathcal{V}[0,T]$ be the set of finite variation paths from $\hat{\Omega}$. Let $f \in C(\hat{\Lambda})$. Then from $\Delta\omega_t = 0$ follows $f(t,\omega) - f(t-,\omega) = 0$, since $d_\infty((t_n,\omega),(t,\omega)) \to 0$ when $t_n \uparrow t$. Hence $f(t,\omega) - f(t-,\omega) \neq 0$ means $\Delta\omega_t \neq 0$.

Definition 1. We say the functional $f \in C([0,T]\times\hat{\Omega})$ is differentiable , if there exist $f^0 \in C([0,T]\times\hat{\Omega})$ and $f^1 \in C([0,T]\times\hat{\Omega})$ such that for all $\omega \in \mathcal{V}$, $f(\cdot,\omega) \in \mathcal{V}$ and

$$\begin{aligned} f(t,\omega) = f(0,\omega) + \int_0^t f^0(s,\omega)ds + \int_0^t f^1(s-,\omega)d\omega_s + \\ + \sum_{s\le t} \big[f(s,\omega) - f(s-,\omega) - f^1(s-,\omega)\Delta\omega_s\big], \end{aligned} \tag{9}$$

for all $(t,\omega) \in [0,T]\times\mathcal{V}$.

A functional f is two times differentiable if f^1 is differentiable, i.e., if there exist $f^{0,1} \in C([0,T]\times\hat{\Omega})$ and $f^{1,1} \in C([0,T]\times\hat{\Omega})$ such that for all $(t,\omega) \in [0,T]\times\mathcal{V}$

$$f^1(t,\omega) = f^1(0,\omega) + \int_0^t f^{1,0}(s,\omega)ds + \int_0^t f^{1,1}(s-,\omega)d\omega_s + V^1(t,\omega), \tag{10}$$

where

$$V^1(t,\omega) = \sum_{s\le t} \big(f^1(s,\omega) - f^1(s-,\omega) - f^{1,1}(s-,\omega)\Delta\omega_s\big).$$

Now we give a genelalization of Theorem 2 from Chitashvili [2] for general cadlag (RCLL) semimartingales.

Theorem 1. *Let f be two times differentiable in the sense of Definition 1 and assume that for some $K > 0$*

$$|f(t,\omega) - f(t-,\omega) - f^1(t-,\omega)\Delta\omega_t| \le K(\Delta\omega_t)^2, \ \ \forall\omega \in \mathcal{V}. \tag{11}$$

Then for any semimartingale X the process $f(t,X)$ is a semimartingale and

$$f(t,X) = f(0,X) + \int_0^t f^0(s,X)ds + \int_0^t f^1(s-,X)dX_s +$$

$$+\frac{1}{2}\int_0^t f^{1,1}(s,X)d\langle X^c\rangle_s + \sum_{s\le t}\big[f(s,X) - f(s-,X) - f^1(s-,X)\Delta X_s\big]. \tag{12}$$

Proof. Let first assume that X is a semimartingale with the decomposition

$$X_t = A_t + M_t, t \in [0,T], \tag{13}$$

where M is a continuous local martingale and A is a process of bounded variation having only finite number of jumps, i.e., the jumps of A are exhausted by graphs of finite number of stopping times $(\tau_i, 1 \le i \le l, l < \infty)$.

Let $X_t^n = A_t + M_t^n$ and

$$M_t(n) = n\int_0^t M_s \exp(-n(\langle M\rangle_t - \langle M\rangle_s)d\langle M\rangle_s. \tag{14}$$

It is proved in [2] that

$$\sup_{s\le t}|M_s(n) - M_t| \to 0, \quad as \ \ n\to\infty, \quad a.s. \tag{15}$$

Since X^n is of bounded variation, f is differentiable and $\Delta X_t^n = \Delta A_t = \Delta X_t$, it follows from (9) that

$$f(t,X^n) = f(0,X) + \int_0^t f^0(s,X^n)ds +$$

$$+\int_0^t f^1(s-,X^n)dX_s + \int_0^t f^1(s-,X^n)d(M_s^n - M_s) +$$

$$+\sum_{s\le t}\big(f(s,X^n) - f(s-,X^n) - f^1(s-,X^n)\Delta X_s\big). \tag{16}$$

Since X admits finite number of jumps, by continuity of f and f^1,

$$\sum_{s\le t}\big(f(s,X^n)-f(s-,X^n)-f^1(s-,X^n)\Delta X_s\big)\to \tag{17}$$

$$\to\sum_{s\le t}\big(f(s,X)-f(s-,X)-f^1(s-,X)\Delta X_s\big)$$

The continuity of f, f^0, f^1 and relation (15) imly that

$$f(t,X^n)\to f(t,X),\quad as\ \ n\to\infty,\quad a.s., \tag{18}$$

$$\int_0^t f^0(s,X^n)ds\to\int_0^t f^0(s,X)ds\quad as\ \ n\to\infty,\quad a.s.. \tag{19}$$

by the dominated convergence theorem and

$$\int_0^t f^1(s-,X^n)dX_s\to\int_0^t f^1(s-,X)dX_s\quad as\ \ n\to\infty,\quad a.s.. \tag{20}$$

by the dominated convergence theorem for stochastic integrals. Here we may use the dominated convergence theorem, since by continuity of $f^i(i=0,1)$ the process $\sup_{n,s\le t}|f^i(s-,X^n)|$ is locally bounded (see Lemma 6 from ([8])).

Let us show now that

$$\int_0^t f^1(s-,X^n)d(M_s^n-M_s)\to\frac{1}{2}\int_0^t f^{1,1}(s,X)d\langle M\rangle_s. \tag{21}$$

Integration by parts and (10) give

$$\int_0^t f^1(s,X^n)d(M_s^n-M_s)=(M_t^n-M_t)f^1(t,X^n)-$$

$$-\int_0^t(M_s^n-M_s)f^{1,0}(s,X^n)ds-\int_0^t(M_s^n-M_s)f^{1,1}(s-,X^n)dA_s$$

$$-\int_0^t(M_s^n-M_s)f^{1,1}(s-,X^n)dX_s^n-\int_0^t(M_s^n-M_s^c)dV^1(s,X^n)=$$

$$=I_t^1(n)+I_t^2(n)+I_t^3(n)+I_t^4(n)+I_t^5(n). \tag{22}$$

$I_t^1(n)\to 0$ (as $n\to\infty$, a.s.) by continuity of f^1 and (15).

$I_t^2(n)$ and $I_t^3(n)$ tend to zero (as $n \to \infty$, a.s.) by continuity of $f^{1,0}$ and $f^{1,1}$, relation (15) and by the dominated convergence theorem (using the same arguments as in (19)-20)).

On the other hand, since A admits finite number of jumps at $(\tau_i, 1 \le i \le l)$

$$I_t(5) = \sum_{s \le t} (M_s^n - M_s)\big(f^1(s, X^n) - f^1(s-, X^n) - f^{1,1}(s-, X^n)\Delta A_s\big) = \quad (23)$$

$$= \sum_{i \le l} (M_{\tau_i}^n - M_{\tau_i})\big(f^1(\tau_i, X^n) - f^1(\tau_i-, X^n) - f^{1,1}(\tau_i-, X^n)\Delta A_{\tau_i}\big) \le$$

$$\le \sup_{s \le t} |M_s^n - M_s| \big(2l \sup_{n, s \le t} |f^1(s, X^n)| + \sup_{n, s \le t} |f^{1,1}(s, X^n)| \sum_{i \le l} |\Delta A_{\tau_i}|\big) \to 0,$$

as $n \to \infty$, since the continuity of $f^1, f^{1,1}$ and relation (15) imply that $\sup_{n, s \le t} |f^1(s, X^n)| + \sup_{n, s \le t} |f^{1,1}(s, X^n)| < \infty$ (a.s.)

Let us consider now the term

$$I_t^4(n) = \int_0^t (M_s - M_s^n) f^{1,1}(s, X^n) dM_s^n$$

Let

$$K_t^n = \int_0^t (M_s - M_s^n) dM_s^n.$$

It is easy to see that ([2])

$$sup_{s \le t} |K_s^n - \frac{1}{2}\langle M \rangle_s| \to 0, \quad as \;\; n \to \infty, \quad a.s \qquad (24)$$

From definition of M^n, using the formula of integration by parts, it follows that M^n admits representation

$$M_t^n = n \int_0^t (M_s - M_s^n) d\langle M \rangle_s.$$

Therefore

$$K_t^n = n \int_0^t (M_s - M_s^n)^2 d\langle M \rangle_s.$$

This implies that K^n is a sequence of increasing processes, which is stochastically bounded by (24) (i.e. satisfies the condition UT from ([7]) and by theorem 6.2 of([7]) (it follows also from lemma 12 of ([3]))

$$\int_0^t (M_s - M_s^n) f^{1,1}(s, X^n) dM_s^n =$$

$$= \int_0^t f^{1,1}(s, X^n) dK_s^n \to \frac{1}{2} \int_0^t f^{1,1}(s, X) d\langle M \rangle_s, \quad n \to \infty,$$

which (together with(22) implies the convergence (21). Therefore, the formula (12) for the process X with decomposition (13) follows by passage to the limit in (16) using relations (17)-(21).

Let consider now the general case. Any semimartingale X admits a decomposition $X_t = A_t + M_t$, where A is a process of finite variation and M is a locally square integrable martingale (such decomposition is not unique, but the continuous martingale parts coincide for all such decompositions of X, which is sufficient for our goals) see ([6]). Let $M_t = M_t^c + M_t^d$, where M^c and M^d are continuous and purely discontinuous martingale parts of M respectively. Let $A_t = A_t^c + A_t^d$ be the decomposition of A, where A^c and A^d are continuous and purely discontinuous processes of finite variations respectively. Note that A^d is the sum of its jumps, whereas M^d is the sum of compensated jumps of M. So, we shall use the decomposition

$$X_t = A_t^c + A_t^d + M_t^c + M_t^d \tag{25}$$

for X and using localization arguments, without loss of generality, one can assume that M^c and M^d are square integrable martingales.

Let $M_t^d(n)$ be the compensated sum of jumps of M of amplitude greater than $1/n$, which is a martingale of finite variation and is expressed as a difference

$$M_t^d(n) = B_t^n - \widetilde{B_t^n}, \tag{26}$$

where $B_t^n = \sum_{s \le t} \Delta M_s I_{(|\Delta M_s| \ge 1/n)}$ and $\widetilde{B^n}$ is the dual predictable projection of B^n. It can be expressed also as compensated stochastic integral (see [5])

$$M_t^d(n) = \int_0^t I_{(|\Delta M_s| > \frac{1}{n})} \overset{\bullet}{C} dM_s,$$

where by $H \overset{\bullet}{C} Y$ we denote the compensated stochastic integral. Since

$$M_t^d(n) - M_t^d = \int_0^t I_{(0 < |\Delta M_s| \le \frac{1}{n})} \overset{\bullet}{C} dM_s,$$

it follows from Doob's inequality and from [5] (theorem 33, Ch.VIII) that

$$E \sup_{s \le t} |M_s^d(n) - M_s^d|^2 \le const E[M^d(n) - M^d]_t = const E[I_{(0 < |\Delta M| \le \frac{1}{n})} \overset{\bullet}{C} M] \le$$

$$\leq const E \int_0^t I_{(0<|\Delta M_s|\leq \frac{1}{n})} d[M]_s \to 0, \quad as \;\; n \to \infty$$

by dominated convergence theorem, since $E[M^d]_T < \infty$. Hence

$$\sup_{s\leq t} |M_s^d(n) - M_s^d| \to 0, \quad as \;\; n \to \infty, \quad a.s. \tag{27}$$

for some subsequence, for which we preserve the same notation.

Let

$$A_t^d(n) = \sum_{s\leq t} I_{(|\Delta A_s|>\frac{1}{n})} \Delta A_s = \int_0^t I_{(|\Delta A_s|>\frac{1}{n})} dA_s.$$

Since

$$|A_t^d - A_t^d(n)| \leq \int_0^t I_{(0<|\Delta A_s|\leq \frac{1}{n})} |dA_s|$$

we have that

$$\sup_{s\leq t} |A_s^d(n) - A_t^d| \to 0, \quad as \;\; n \to \infty, \quad a.s. \tag{28}$$

Let

$$X_t^n = A_t^c + A_t^d(n) + M_t^d(n) + M_t^c.$$

Relations (27) and (28) imply that

$$\sup_{s\leq t} |X_s(n) - X_s| \to 0, \quad as \;\; n \to \infty, \quad a.s., \tag{29}$$

Thus X^n is a sum of continuous local martingale M^c and a process of finite variation $A_t^c + A_t^d(n) + M_t^d(n)$ which admits only finite number of jumps for every $n \geq 1$.

Therefore, as it is already proved,

$$f(t, X^n) = f(0, X^n) + \int_0^t f^0(s, X^n) ds + \int_0^t f^1(s-, X^n) dX_s +$$

$$+ \int_0^t f^1(s-, X^n) d(M_s^n(d) - M_s^d) + \int_0^t f^1(s-, X^n) d(A_s^n(d) - A_s^d)$$

$$+ \frac{1}{2} \int_0^t f^{1,1}(s, X) d\langle X^c \rangle_s +$$

$$+ \sum_{s\leq t} \big(f(s, X^n) - f(s-, X^n) - f^1(s-, X^n) \Delta X_s^n \big). \tag{30}$$

By continuity and boundedness of f, f^0 and f^1

$$f(t, X^n) \to f(t, X), \quad as \;\; n \to \infty, \quad a.s., \tag{31}$$

$$\int_0^t f^0(s, X^n)ds \to \int_0^t f^0(s, X)ds \quad as \;\; n \to \infty, \quad a.s.. \tag{32}$$

by the dominated convergence theorem and

$$\int_0^t f^1(s-, X^n)dX_s \to \int_0^t f^1(s-, X)dX_s \quad as \;\; n \to \infty, \quad a.s.. \tag{33}$$

by the dominated convergence theorem for stochastic integrals (using the same arguments as in (19)-20)).

By properties of compensated stochastic integrals

$$\int_0^t f^1(s-, X^n)d(M_s^d(n) - M_s^d) = \int_0^t f^1(s-, X^n)I_{(0<|\Delta M_s|\le\frac{1}{n})}\overset{\bullet}{C}dM_s$$

and using theorem 33, Ch. VIII from [5]

$$E\Big(\int_0^t f^1(s-, X^n)I_{(0<|\Delta M_s|\le\frac{1}{n})}\overset{\bullet}{C}dM_s\Big)^2 \le$$

$$\le const E\int_0^t (f^1(s-, X^n))^2 I_{(0<|\Delta M_s|\le\frac{1}{n})}d[M^d]_s \to 0 \quad as \;\; n \to \infty \tag{34}$$

by dominated convergence theorem, since $\sup_{n,s\le t}(f^1(s, X^n))^2$ is locally bounded, $I_{(0<|\Delta M_s|\le\frac{1}{n})} \to 0$ and $E[M^d]_T < \infty$.

Similarly, $\int_0^t f^1(s-, X^n)d(A_s^n(d) - A_s^d)$ also tends to zero, since

$$\int_0^t f^1(s-, X^n)d(A_s^n(d) - A_s^d) \le \int_0^t |f^1(s-, X^n)|I_{(0<|\Delta A_s|\le\frac{1}{n})}|dA_s| \to 0. \tag{35}$$

From (26)

$$\Delta M_s^n(d) = \Delta M_s I_{(|\Delta M_s|\ge 1/n)} - \big(\Delta M I_{(|\Delta M|\ge 1/n)}\big)^p_s,$$

where Y^p is the usual projection of Y. Here we used the fact that the jump of the dual projection of B^n is the usual projection of the jump, i.e. $\Delta\widetilde{B_t^n} = (\Delta B^n)_t^p$. Therefore, using condition (11) we have that

$$|(f(s, X^n) - f(s-, X^n) - f^1(s-, X^n)\Delta X_s^n| \le cost.(\Delta X_s^n)^2 =$$

$$= const.\big(\Delta A_s I_{(|\Delta A_s|\geq 1/n)} + \Delta M_s I_{(|\Delta M_s|\geq 1/n)} - (\Delta M I_{(|\Delta M|\geq 1/n)})^p_s\big)^2 \leq$$
$$\leq 3const.\big((\Delta A_s)^2 + (\Delta M_s)^2 + E((\Delta M_s)^2/F_{s-})\big). \qquad (36)$$

Since, it follows from (29) and continuity of f and f^1

$$f(s,X^n)-f(s-,X^n)-f^1(s-,X^n)\Delta X^n_s \to f(s,X)-f(s-,X)-f^1(s-,X)\Delta X_s$$

and

$$\sum_{s\leq t}\big((\Delta A_s)^2 + (\Delta M_s)^2 + E((\Delta M_s)^2/F_{s-})\big) \leq [X,X]_t + \langle M,M\rangle_t < \infty$$

the dominated convergence theorem implies that

$$\sum_{s\leq t}\big(f(s,X^n) - f(s-,X^n) - f^1(s-,X^n)\Delta X^n_s\big) \to$$

$$\to \sum_{s\leq t}\big(f(s,X) - f(s-,X) - f^1(s-,X)\Delta X_s\big), \quad as \quad n\to\infty. \qquad (37)$$

Therefore, passing to the limit in (30) it follows from (31)-(37) that (12) holds.

3 The relations between various definitions

Let $\Omega := \{\omega \in C([0,T],\mathbb{R}^d) : \omega_0 = \mathbf{0}\}$ be the set of continuous paths with initial value $\mathbf{0}$, B the canonical process, $\mathbb{F}$ the filtration generated by B.

Following Dupire [4] we define time and space derivatives, called also horisontal and vertical derivatives, of the non-anticipated functionals.

Definition 2.

A non-anticipative functional $f(t,\omega)$ is said to be horisontally differentiable at $(t,\omega)\in\Lambda_T$ if

$$\partial_t u(t,\omega) := \lim_{h\to 0, h>0}\frac{1}{h}\big[u(t+h,\omega^t) - u(t,\omega)\big], \qquad t<T, \qquad (38)$$

exists. If $\partial_t u(t,\omega)$ exists for all $(t,\omega)\in\Lambda_T$, then the non-anticipating functional ∂f_t is called the horisontal derivative of f.

A non-anticipative functional $f(t,\omega)$ is vertically differentiable at $(t,\omega)\in\Lambda_T$ if

$$\partial_{\omega_i} u(t,\omega) := \lim_{h\to 0}\frac{1}{h}\big[u(t,\omega + h1_{[t,T]}e_i) - u(t,\omega)\big], \qquad (39)$$

exists. If f is vertically differentiable at all $(t,\omega) \in \Lambda_T$ then the map $\partial_\omega f(t,\omega) \to R$ defines a non-anticipated map called the vertical derivative of f.

Similarly one can define

$$\partial_{\omega_i\omega_j} u := \partial_{\omega_i}(u_{\omega_j}), \qquad i,j = 1,\dots,d. \tag{40}$$

Define $C^{1,k}([0,T))$ as the set of functionals F, which are

- horizontally differentiable with $\partial_t F$ continuous at fixed times,
- k times vertically differentiable with continuous $\partial_{\omega_{i_1}}...\partial_{\omega_{i_l}} F$ for $(i_1,...,i_l) \in \{1,...,d\}^l,\ l = 1,...,k$.

The following assertion follows from the generalized Itô formula for non-anticipating functionals proved in [3] (see also theorem 8 from [8]).

Theorem 2. *Let $f \in C^{1,1}([0,T] \times \hat{\Omega})$. Then for all $(t,\omega) \in [0,T] \times \mathcal{V}$*

$$f(t,\omega) = f(0,\omega) + \int_0^t \partial_t f(s,\omega)ds + \int_0^t \partial_\omega f(s-,\omega)d\omega_s + \sum_{s\le t}(f(s,\omega) - f(s-,\omega) - \partial_\omega f(s-,\omega)\Delta\omega_s)$$

and $f(t,\omega) \in \mathcal{V}$ for all $\omega \in \mathcal{V}$.

Corollary. If $f \in C^{1,1}([0,T] \times \hat{\Omega})$, then f is differentiable in the sense of Definition 1 and

$$\partial_t f = f^0, \quad \partial_\omega f = f^1.$$

In order to compare Dupire's derivatives with Chitashvili's derivative (the derivative in the sense of Definition 1), we introduce another type of vertical (or vertical-diagonal) derivative where, unlike to Dupire's derivative $\partial_\omega f$, the path deformation (or perturbation) of continuous paths remain continuous.

We say that a non-anticipative functional $f(t,\omega)$ is differentiable and denote this differential by $D_\omega f(t,\omega)$, if the limit

$$D_\omega F(t,\omega) := \lim_{h\to 0, h>0} \frac{f(t+h,\omega^t + \chi_{t,h}) - f(t+h,\omega^t)}{h}, \tag{41}$$

exists for all $(t,\omega) \in [0,T] \times V$, where

$$\chi_{t,h}(s) = (s-t)1_{(t,t+h]}(s) + h1_{(t+h,T]}(s).$$

Proposition 1. *Let $f \in C([0,T]\times\hat{\Omega})$ be differentiable in the sense of Definition 1, i.e., there exist $f^0, f^1 \in C([0,T]\times\hat{\Omega})$, such that for all $(t,\omega)\in[0,T]\times\mathcal{V}$*

$$f(t,\omega) = f(0,\omega) + \int_0^t f^0(s,\omega)ds + \int_0^t f^1(s-,\omega)d\omega_s + V(t,\omega), \tag{42}$$

where

$$V(t,\omega) := \sum_{s\le t}\left[f(s,\omega) - f(s-,\omega) - f^1(s-,\omega)\Delta\omega_s\right]$$

is of finite variation for all $\omega\in\mathcal{V}$.

Then for all $(t,\omega)\in[0,T]\times\mathcal{V}$

$$f^0(t,\omega) = \partial_t f(t,\omega) \quad \textit{and} \quad f^1(t,\omega) = D_\omega f(t,\omega). \tag{43}$$

Proof. Since ω^t is constant on $[t,T]$ and $f(t,\omega^t) = f(t,\omega)$, if $s\le t$, from (42) we have that

$$f(t+h,\omega^t) = f(0,\omega) + \int_0^t f^0(s,\omega)ds + \int_t^{t+h} f^0(s,\omega^t)ds + \tag{44}$$
$$+ \int_0^t f^1(s-,\omega)d\omega_s + V(t,\omega)$$

and

$$f(t+h,\omega^t+\chi_{t,h}) = f(0,\omega) + \int_0^t f^0(s,\omega)ds + \int_0^t f^1(s-,\omega)d\omega_s + \tag{45}$$
$$+ \int_t^{t+h} f^0(s,\omega^t+\chi_{t,h})ds + \int_t^{t+h} f^1(s,\omega^t+\chi_{t,h})ds + V(t,\omega).$$

Therefore

$$\frac{f(t+h,\omega^t) - f(t,\omega)}{h} = \frac{1}{h}\int_t^{t+h} f^0(s,\omega^t)ds \to f^0(t,\omega)$$

by continuity of f^0.

It is evident tha $t\chi_{t,h}(s)\le h$ and

$$\frac{\chi_{t,h}(s) - \chi_{t,0}(s)}{h} = \frac{\chi_{t,h}(s)}{h} \to 1_{[t,T]}(s) \; as \; h\to 0+, \; \forall s\in[0,T].$$

Trerefore, relations (45)-(44) and continuity of f^1 and f^0 imply that

$$\frac{f(t+h,\omega^t+\chi_{t,h})-f(t+h,\omega^t)}{h}=\frac{1}{h}\int_t^{t+h}\big(f^0(s,\omega^t+\chi_{t,h})-f^0(s,\omega^t)\big)ds$$
$$+\frac{1}{h}\int_t^{t+h}f^1(s,\omega^t+\chi_{t,h})ds\to f^1(t,\omega),\ as\ h\to 0+.$$

Corrolary. Let $f\in C^{1,1}([0,T]\times\hat{\Omega})$. Then

$$\partial_\omega f(t,\omega)=f^1(t,\omega)=D_\omega f(t,\omega)=\lim_{h\to 0,h>0}\frac{f(t+h,\omega^t+\chi_{t,h})-f(t+h,\omega^t)}{h}$$
$$\forall(t,\omega)\in[0,T]\times\mathcal{V}.$$

Counterexample 1. Let $g\in C_b'(\mathbb{R})$ and $f(t,\omega)=g(\omega_t)-g(\omega_{t-})$. Then $\partial_\omega f(t,\omega)=g'(\omega_t)$ and

$$\lim_{h\to 0+}\frac{f(t+h,\omega^t+\chi_{t,h})-f(t+h,\omega^t)}{h}=0,\ \text{since}$$
$$f(t+h,\omega^t+\chi_{t,h})-f(t+h,\omega^t)=g(\omega_t+h)-g(\omega_t+h)-g(\omega_t)+g(\omega_t)=0.$$

Clearly that $f\bar{\in}C_b(\hat{\Lambda})$ and $\partial_\omega f\in C_b(\hat{\Lambda})$. $\partial_t f=\infty$.

Proposition 2. *Let for $f\in C([0,T]\times\hat{\Omega})$ be differentiable in the sense of Definition 1 and*

$$\big|f(t,\omega)-f(t-,\omega)-\Delta\omega_t f^1(t-,\omega)\big|\le K|\Delta\omega_t|^2 \tag{46}$$

for some $K>0$. Then

$$f^0(t,\omega)=\partial_t f(t,\omega),\quad \forall(t,\omega)\in\Lambda,$$
$$f^1(t,\omega)=\partial_\omega f(t,\omega),\quad \forall\omega\in C([0,T])$$

(or for all ω continuous at t).

Proof. For $\omega\in D([0,T])$ let $\tilde{\omega}=\omega_s$ if $s<t$ and $\tilde{\omega}=\omega_s+h$, if $s\ge t$, i.e. $\tilde{\omega}=\omega^{t-}+h1_{[t,T]}$.

Then using condition (46) for $\tilde{\omega}$ we have

$$\left|\frac{f(t,\omega^{t-}+h1_{[t,T]})-f(t_-,\omega)}{h}-f^1(t-,\omega)\right|\le K|h|,\ \forall h.$$

It follows from here that

$$\lim_{h\to 0}\frac{f(t,\omega^{t-}+h1_{[t,T]})-f(t_-,\omega)}{h}=f^1(t-,\omega),$$

which implies that $f^1(t,\omega)=\partial_\omega f(t,\omega)$ if ω is continuous at t.

Equality $f^1(t,\omega)=\partial_t f(t,\omega),\forall(t,\omega)\in\Lambda$ is proved in Proposition 1.

Proposition 3. *Let there exist $f_t, f_\omega \in C(\Lambda)$, such that*

$$f_t(t,\omega)=\lim_{h\to 0,h>0}\frac{f(t+h,\omega^t)-f(t,\omega)}{h},$$
$$f_\omega(t,\omega)=\lim_{h\to 0,h>0}\frac{f(t+h,\omega)-f(t+h,\omega^t)}{\omega_{t+h}-\omega_t},$$
$$\forall(t,\omega)\in[0,T]\times C^1([0,T]).$$

Then $\forall(t,\omega)\in[0,T]\times C^1([0,T])$

$$f(t,\omega)-f(0,\omega)=\int_0^t f_t(s,\omega)ds+\int_0^t f_\omega(s,\omega)d\omega_s. \tag{47}$$

Proof. We have

$$\lim_{h\to 0,h>0}\frac{f(t+h,\omega)-f(t,\omega)}{h}$$
$$=\lim_{h\to 0+}\frac{f(t+h,\omega)-f(t+h,\omega^t)}{\omega_{t+h}-\omega_t}\times\frac{\omega_{t+h}-\omega_t}{h}$$
$$+\lim_{h\to 0+}\frac{f(t+h,\omega^t)-f(t,\omega)}{h}=\omega'(t)f_\omega(t,\omega)+f_t(t,\omega),$$
$$\forall(t,\omega)\in[0,T]\times C^1([0,T]).$$

Hence right derivative of

$$f(t,\omega)-f(0,\omega)-\int_0^t f_t(s,\omega)ds-\int_0^t f_\omega(s,\omega)\omega'_s ds$$

is zero for each $\omega\in C^1$. By the Lemma A1 of appendix formula (47) is satisfied.

Theorem 3. *Let $f_t, f_\omega \in C(\Lambda)$ be such that*

$$\lim_{h\to 0,h>0} \frac{f(t+h,\omega^t)-f(t,\omega)}{h} = f_t(t,\omega),$$
$$\lim_{h\to 0,h>0} \frac{f(t+h,\omega)-f(t+h,\omega^t)}{h} = \omega'(t)f_\omega(t,\omega),$$
$$\forall (t,\omega) \in [0,T]\times C'[0,T].$$

Assume also that $f(t,\omega) \in \mathcal{V}$ for any $\omega \in \mathcal{V}$. Then

$$f(t,\omega) = f(0,\omega) + \int_0^t f_t(s,\omega)ds + \int_0^t f_\omega(s,\omega)d\omega_s^c + \sum_{s\le t} f(s,\omega) - f(s-,\omega).$$

Proof. For $\omega \in V$ we have $\omega = \omega^c + \omega^d,\ \omega^d = \sum_{s\le t}\Delta\omega_s,\ \omega^c \in C$. Set

$$\omega^{d,n} = \sum_{s\le t, |\Delta\omega_s|>\frac{1}{n}} \Delta\omega_s,\ \omega^n = \omega^c + \omega^{d,n}.$$

It is evident that as $n \to 0$

$$|\omega^n - \omega|_T = |\omega^d - \omega^{d,n}|_T = \max_t |\int_0^t 1_{(|\Delta\omega_s|\le\frac{1}{n})}d\omega_s^d \le \int_0^T 1_{(|\Delta\omega_s|\le\frac{1}{n})}dvar_s(\omega^d) \to 0.$$

We know that discontinuity points of f are also discontinuity points of ω. Let $\{t_1 < ... < t_k\} = \{s : |\Delta\omega_s| > \frac{1}{n}\} \cup \{0,T\}$. Suppose that $\omega^\varepsilon \in C'$ is differentiable approximation of such that $\omega^c,\ var_T(\omega^\varepsilon - \omega^c) < \varepsilon$ and let $\omega^{n,\varepsilon} = \omega^\varepsilon + \omega^{d,n}$. Then by the Proposition 3

$$f(t,\omega^{n,\varepsilon}) - f(t_i,\omega^{n,\varepsilon}) - \int_{t_i}^t f_t(s,\omega^{n,\varepsilon})ds - \int_{t_i}^t f_\omega^{n,\varepsilon}(s,\omega^{n,\varepsilon})\omega_s^{'\varepsilon}ds = 0,\ t \in [t_i,t_{i+1})$$

and

$$f(T,\omega^{n,\varepsilon})-f(0,\omega^{n,\varepsilon})=\sum_{i\geq 1} f(t_i,\omega^{n,\varepsilon})-f(t_{i-1},\omega^{n,\varepsilon})$$

$$=\sum f(t_i-,\omega^{n,\varepsilon})-f(t_{i-1},\omega^{n,\varepsilon})+\sum f(t_i,\omega^{n,\varepsilon})-f(t_i-,\omega^{n,\varepsilon})$$

$$=\sum\int_{t_{i-1}}^{t_i} f_t(s,\omega^{n,\varepsilon})ds+\int_{t_{i-1}}^{t_i} f_\omega(s,\omega^{n,\varepsilon})\omega_s^{'\varepsilon}ds+\sum f(t_i,\omega^{n,\varepsilon})-f(t_i-,\omega^{n,\varepsilon})$$

$$=\sum\int_{t_{i-1}}^{t_i} f_t(s,\omega^{n,\varepsilon})ds+\int_{t_{i-1}}^{t_i} f_\omega(s,\omega^{n,\varepsilon})d\omega_s^{n,\varepsilon}$$

$$-f_\omega(t_i-,\omega^{n,\varepsilon})\Delta\omega_{t_i}^{n,\varepsilon}+\sum f(t_i,\omega^{n,\varepsilon})-f(t_i-,\omega^{n,\varepsilon})$$

$$=\int_0^T f_s(s,\omega^{n,\varepsilon})ds+\int_0^T f_\omega(s,\omega^{n,\varepsilon})d\omega_s^{n,\varepsilon}$$

$$+\sum f(t_i,\omega^{n,\varepsilon})-f(t_i-,\omega^{n,\varepsilon})-f_\omega(t_i-,\omega^{n,\varepsilon})\Delta\omega_{t_i}^{n,\varepsilon}$$

$$=\int_0^T f_s(s,\omega^{n,\varepsilon})ds+\int_0^T f_\omega(s,\omega^{n,\varepsilon})d\omega_s^{\varepsilon}+\sum f(t_i,\omega^{n,\varepsilon})-f(t_i-,\omega^{n,\varepsilon}).$$

Passing to the limit as $\varepsilon\to 0$ we get

$$f(T,\omega^n)-f(0,\omega^n)$$

$$=\int_0^T f_s(s,\omega^n)ds+\int_0^T f_\omega(s,\omega^n)d\omega_s^c+\sum f(t_i,\omega^n)-f(t_i-,\omega^n).$$

By the continuity and boudendness on the compact set $[0,T]\times\{\omega,\omega^n,n=1,2,...\}$ of functions $f,\ f_t,\ f_\omega$

$$f(t,\omega^n)\to f(t,\omega),\ \int_0^t f_s(s,\omega^n)ds\to\int_0^t f_s(s,\omega)ds,$$

$$\int_0^t f_s(s,\omega^n)d\omega_s^c\to\int_0^t f_s(s,\omega)d\omega_s^c,\ as\ n\to\infty.$$

It remains to show convergence of sum. Since $f^d(t,\omega)=\sum_{s\leq t} f(s,\omega)-f(s-,\omega)$ is finite variation

$$\sum f(t_i,\omega^n)-f(t_i-,\omega^n)-\sum f(t_i,\omega)-f(t_i-,\omega)$$

$$=\sum_{s\leq t}(f(s,\omega)-f(s-,\omega))1_{(|\Delta\omega_s|\leq\frac{1}{n})}$$

$$=\int_0^t 1_{(|\Delta\omega_s|\leq\frac{1}{n})}df^d(s,\omega)\to 0,\ as\ n\to\infty.$$

The last convergence follows from the dominated convergence theorem.

4 Appendix

Lemma A1. Let f be a real-valued, continuous function, defined on an arbitrary interval I of the real line. If f is right (or left) differentiable at every point $a \in I$, which is not the supremum (infimum) of the interval, and if this right (left) derivative is always zero, then f is a constant.

Proof.

For a proof by contradiction, assume there exist $a < b$ in I such that $f(a) \neq f(b)$. Then

$$\varepsilon := \frac{|f(b) - f(a)|}{2(b-a)} > 0.$$

Define c as the infimum of all those x in the interval $(a, b]$ for which the difference quotient of f exceeds ε in absolute value, i.e.

$$c = \inf\{\, x \in (a, b] \mid |f(x) - f(a)| > \varepsilon(x - a) \,\}.$$

Due to the continuity of f, it follows that $c < b$ and $|f(c) - f(a)| = \varepsilon(c - a)$. At c the right derivative of f is zero by assumption, hence there exists d in the interval $(c, b]$ with $|f(x) - f(c)| \leq \varepsilon(x - c)$ for all $x \in (c, d]$. Hence, by the triangle inequality,

$$|f(x) - f(a)| \leq |f(x) - f(c)| + |f(c) - f(a)| \leq \varepsilon(x - a)$$

for all x in $[c, d)$, which contradicts the definition of c.

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