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შინაარსი

EMBODIMENT SYMBOLS IN ORGANIZATIONAL ANTHROPOLOGY

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ABSTRACT

The paper presents some reflections from the organizational anthropology view on the processes in organizations nowadays, meeting the challenges of remote work, shift of management process from the traditional office to the abstract virtual area. The paper presents the results of the sample research, interpreting them in the five organizational symbols framework. There are made some conclusions on the symbols embodiment in organizational culture and their impact on the involvement not as an attitude but as a value in nowadays economy of participation.

Key words*: organizational anthropology, organizational symbols, organizational culture*

> Culture describes social action as depending on the meaning it has for those involved, while social structure describes social action from the point of view of its consequences on the functioning of the social system.
>
> M. Alvesson

Descartes declared, 'I think therefore I am', and it was a transition in the Western thought, the proof of conscious humankind. At the same time, it was the move to dis-embodiment, the clear separation of mind, and a mind's rip out of somatic unity of mind, body, soul and spirit (soma means here a living process). Phenomenology (traced to Hegel, Nietzsche, Husserl, and later to Merleau-Ponty) moved the focus from the battle for objective reality onto the subjective experience, awareness and actions from the specific awareness degree. George G.Lakoff clearly

underlined the importance of focusing on the 'filters' in human reasoning, as "Real human beings are not, for the most part, in conscious control of-or even consciously aware of-their reasoning. Most of their reason, besides, is based on various kinds of prototypes, framings, and metaphors" (Lakoff, 1999, p. 16). Phenomenology questioned the nature of cognition, and stated the principle 'I am therefore I feel how I am thinking on something'. This key moment of finding out the feeling how one thinks about something, makes organizational anthropology studies to move from the principle 'what is' to the paradigm 'how it is'. Probably, the only connection with this aspect of economic relationships in organizations can be set though the symbols, specifically, one of the symbol's display, - metaphor.

Organization is a mini-sample of the culture and society, often is displayed as a prototype. Nowadays humankind is moving through the transformation of some values, many processes are intervened, and the old approaches fail to help organizations operate, perform and get results. There is a need of some integrated approach in managing the subjective context of embodiment. The economy changes from the economy of observing towards the economy of involvement. People want to do many things themselves, they want to make pictures, draw, write stories, be influencers and motivational speakers, do their own blogs, online schools and businesses. Economy of active participation is based on the idea of embodiment, - building awareness and mindfulness. Employees with a greater attention want to find the sense in what they do and see more options for the choice. This active involvement in everything they do, make organizations to remote not only some work, because of the pandemic, but as well some managerial functions. The absence of the traditional office and some basic rituals (for example, greeting security in the morning, or going to lunch) made the challenge of involvement more obvious for organizations.

There is a shift in effectiveness from adaptation (orientation programs) to involvement (active sharing of some values and metaphors). In this shift more obvious is a need for symbols to be integrated in the organizational social and cultural processes. High level of involvement and

stronger identity will be established as a consequence of the pandemic influence we are experiencing now. Organizational culture is a symbolic frame for embodiment of the sense, choice, awareness of the job, or of work in general.

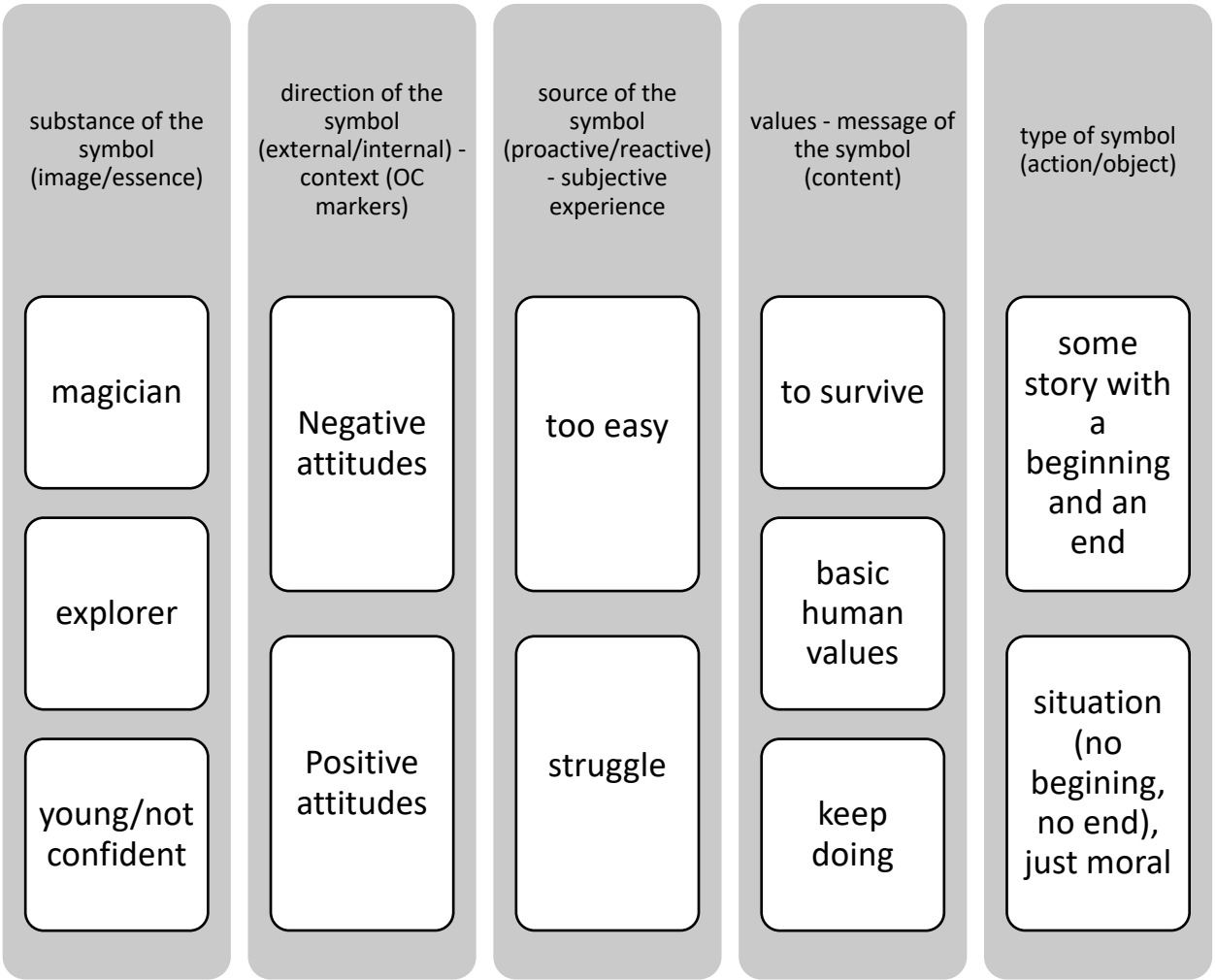
In humankind history retrospective homo sapiens could happen because of the desire to cognize and symbolize. There appeared art, science, philosophy, and culture as over-biological codes, opening the perspectives for symbolic reflection. The symbolic production in organization is explored within the context of metasymbolic, - when some forms symbolizes other forms which are symbolic as well, as stated by K.Skoldberg (Alvesson, 2002). It was possible because of the metaphor, the 'filter' of objective reality and subjective perspective. The metaphor as well can be shared, and organizational culture is a kind of shared embodied metaphor, though which employees 'learn' their relationships. Usually organizational culture cannot be shaped directly from management, it comes from the integration of employees' behavior, activities, values and beliefs, and can be incorporated into the identity and job involvement. Managers work with these symbolic forms, even if they do not consciously realize that. From once side they deal with them in themselves, from another, - in doing things through and with their employees. Attitudes, in-group relations, perception, emotions, motivation, and other processes cannot be managed directly. One cannot just tell employee to become happy, or change the negative attitude. It is possible only within the context of transformation, and change in people do not happen only from the external factors, it is a combination of situational influence and inner energy of change, like interactionist perspective in behaviorism.

S.R.Fuller describes organizational symbolism as the study of the expressive functions of organizational life (Fuller, 2011). The thinker offers the following five dimensions as organizational symbols: type of symbol (action/object), direction of the symbol (external/internal), source of the symbol (proactive/reactive), message of the symbol (content), substance of the symbol (image/essence). These dimensions are integrated into the metaphoric

interpretation of the employment experience from the respondents. The respondents were offered to write a fairy-tale of their work experience, following some coaching questions. 35 respondents (from Georgian, Russian and international business organizations), describing their work embodiment experience, for the main character (hero) mostly appeal to three basic types: magician, explorer or someone young (not experienced).

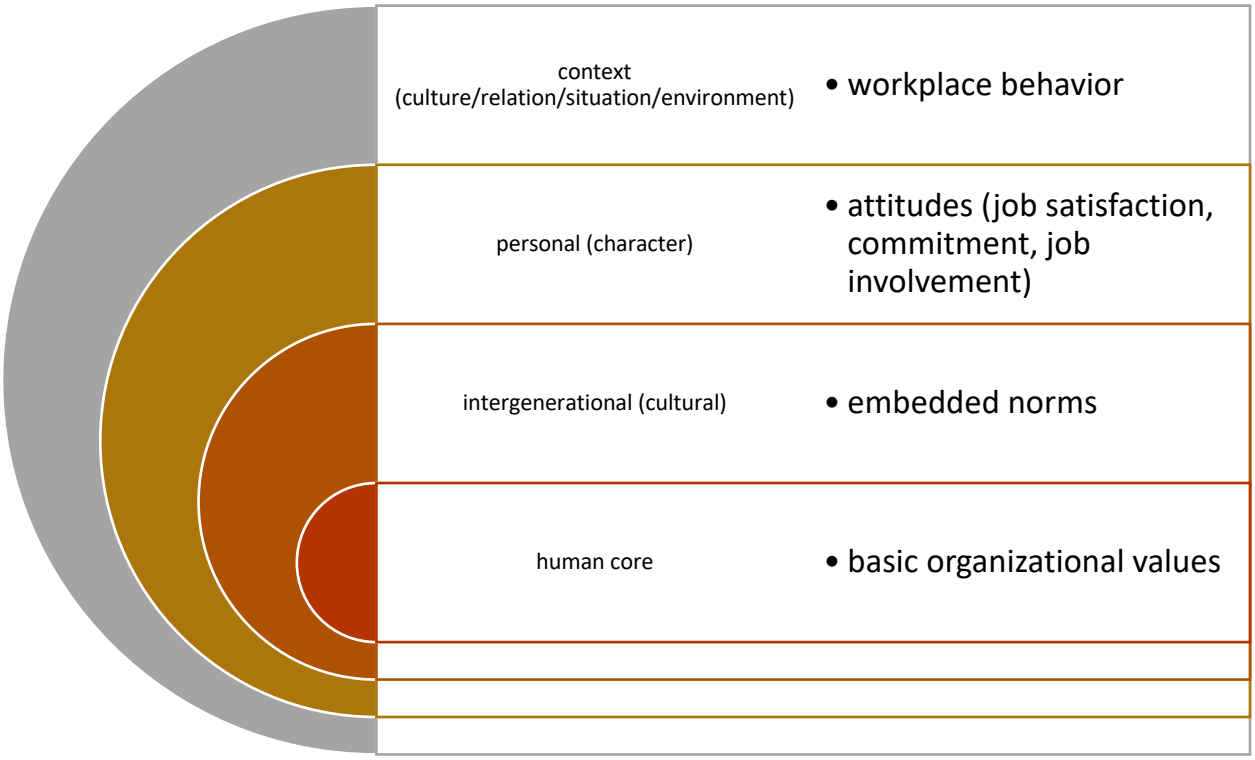
It underlines the importance of involvement (or even identification with the work), as mentioned above. Following the five Fuller's dimensions, we can build the table of meta-metaphors from the survey (See Exhibit 1).

Exhibit 1:



The behavior patterns are formed by several layers, like human core (something we all have in common from homo sapiens sapiens), the intergenerational patterns we inherit, some personal trait sin character. These patterns are displayed in relational, work, cultural context in the following correlation in layers (se Exhibit 2).

Exhibit 2:



Organizational culture is an integrated field that include all these elements, it is not just the cultural context, it an integrated 'dojo' for behavioral and communicative patterns, and the quality, depth and intensity of embodiment is defined by the symbols' strength. Organizational culture is based on the symbolic meanings defining humankind, like primary sense, as Yuval Harari states in his book "Sapiens" (Harari, 2015), Homo Sapiens is transcended the biological survival limits, breaking the laws of natural selection, and starting the laws of intelligent

designs. People is an important element in understanding organization, along with structure and purpose, so human capital is the most dynamic and invaluable resource of the organization. Moreover, involvement, engagement, participation are already not the attitudes, but important values that matter in organizational survival and new metaphors formulation.

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HUMAN RESOURCE MANAGEMENT PHILOSOPHY IN TODAY'S ORGANIZATION

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ABSTRACT

The main essence of the topic - is human resources, which reflects the wealth of any society and for the development it is possible to produce, improve and utilize the same resources, taking into consideration the interests of each individual. Human views are based on his knowledge, qualifications and experience. This also refers to the management of human resources. Any decision regarding the personnel, which is taken by the management, is derived from the individual perception of human resources management. As the experience has shown, in terms of global competition the success is achieved by the managers, who can better adjust to variable conditions. This, in turn, requires the existence of the loyal and qualified employees as well.

The human resource manager's profession is significantly different from traditional staff work. It is related to the integration and successful implementation of the organization's goals. The important stage of this process is assimilation of the idea of system approach, not only functional but also an organization as a system and Human Resource Management was formed based on new different models. People represent important resources of the organization, they determine the future strategy of the company, and the strategy itself is realized by people. The success of the chosen strategy by the organization depends not only on the decisions taken in the past but also for the effective implementation of these decisions by the people employed in the organization. That's why it is important for the company to have clearly defined employee function. Creating the value of the company is rapidly increasing the role of human

resources and corporate culture as intangible assets. More specifically, these intangible assets are a decisive factor for the establishment of the value of the company.

The management, in general - is the process of rational and effective use of the resources which is achieved through planning, organizing, leading and control. Issues such as cash flow, competition, and revenue growth are top of mind for small business owners and their teams. Along with these challenges comes another area many companies struggle with human resource management. Human resource management (HRM) includes: Job design and analysis, Workforce planning, Training and development, Performance management, Compensation and benefits, Legal issues and etc.

HRM can be a challenge for small businesses especially, which typically don't have an HR department to rely on. They may be limited to one HR person, or this responsibility may still belong to the CEO. Regardless, small business owners need to understand the challenges facing them, so they're prepared to tackle HR issues as their company, and workforce grows:

- Compliance with Laws and Regulation
- Workforce Training and Leadership Development
- Adapting to Innovation
- Compensation and Benefits Packages
- Managing Talented Employees
- Workplace Diversity Issues

Above mentioned topics are today's most common human resource challenges along with solutions you can quickly implement in your business. HRM can be a challenge for small businesses especially, which typically don't have an HR department to rely on. They may be limited to one HR person, or this responsibility may still belong to the CEO. Regardless, small business owners need to understand the challenges facing them, so they're prepared to tackle HR issues as their company, and workforce, grows.

It’s important to know common HR issues so you can put the right policies and procedures in place now. Understanding the complexities of employee benefits, employment laws, leadership development, and other areas will help you stay ahead of the competition and meet your business goals.

Humans have always been complex, and having to manage people has always been tricky. Now, the digital age has added even more complexity to human resource management, like the transparency of social media, the persistence of software updates, and the remoteness of international teams. The modern HR technology term, human capital management (HCM), has come into more frequent use than the term, HRM, with the widespread adoption by large and midsize companies and other organizations of software to manage many HR functions.

Behind production of every product or service there is a human mind, effort and man hours (working hours). No product or service can be produced without help of human being. Human being is fundamental resource for making or construction of anything. Every organization desire is to have skilled and competent people to make their organization competent and best.

It is undisputed fact that humans are being replaced by artificial intelligence which are in the form of robots. But all jobs cannot be handed over to Robots, to say in other words robots have its own limitations and all roles cannot be handled by robots. Though British theoretical physicist Stephen Hawking, Cambridge professor expressed about destruction of middle-class jobs due to raise of artificial intelligence, he still felt that natural intelligence or need for application of human mind is inevitable in certain roles.

The normative perspective on Human Resource Management deals with HRM from two basic views: "hard HRM" and "soft HRM". "Hard" HRM embraces all those elements in employment relations laying emphasis on employee's compliance, quantitative output, managers, task and the development of the organization. "Soft" HRM will tend to favor flexibility, negotiation,

performance, quality, recognition of environments and rights in employment relations. It is more strategic and long term.

The critical perspective of HRM is an outcome of normative perception. It proposes that organizations maintain their "soft HRM" approach only to show in their policies but in reality, they practice "hard HRM" to extend management control.

They pretend to be concerned for workers and exploit them through work intensification and downsizing. Thus, Critical Perspective proposes that HRM has only changed organizational rhetoric and reality has not changed since the introduction of Personnel. However, it also argues that HRM uses a unitary, soft HRM rhetoric to obscure hard reality characterized by increased management control and diminished job security for employees. The first proposition describes HRM as powerless and the second as powerful.

Behavioral perspective of HRM believes that it is vital for an organization to control the behavior of its employees to bring the desired results from them. Focus is on the identification of desired behavior, ensuring availability of opportunities and environment for desired behavior, developing employees' skills to bring desired behavior, and motivating employees to behave as desired.

Strategic perspective of HRM believes that the human resources are valuable in improving an organization’s efficiency or effectiveness. It provides a strategic framework to support long term business goals and objectives. The focus is on longer term people issues and macro concerns about structure, quality, culture, values, commitment and matching resources to future need.

One of the factors behind organizations giving a lot of attention to their people is the nature of the firms in the current business environment. Given the fact that there has been a steady movement towards an economy based on services, it becomes important for firms engaged in the service sector to keep their employees motivated and productive. Even in the

manufacturing and the traditional sectors, the need to remain competitive has meant that firms in these sectors deploy strategies that make effective use of their resources. This changed business landscape has come about as a result of a paradigm shift in the way businesses and firms view their employees as more than just resources and instead adopt a "people first" approach.

The development of the HR Management Philosophy is a long-term process. The philosophy is usually informal and respects values and opinions of the main stakeholders. The leader of the organization has a significant influence on the HR Philosophy. The HR Leader is the next one influencing the philosophy significantly.

The modern leader usually requires a modern HR Management approach and builds the environment suitable for the evolution of the modern HR Management. This is the HR Management Philosophy behind the scene.

The main factors influencing the HR philosophy in the organization are:

- Leadership Style
- Corporate Culture
- Corporate Values
- Market Competition

The leadership style is extremely important. The leaders of the organization are the role models for managers and employees. The manager always tries to act as the leader does. The behavior is observed and spread across the entire organization. The leader sets the basic expectations from all employees and managers. They adjust their behavior to be fully compliant with the leader's expectations. The HR Management Philosophy is the same story.

The organizations using the productivity-based configuration, performance oriented processes were used within each HR practice to maximize productivity but were not used to enhance integration between HR practices. As a result, each HR practice operated more or less as a stand-alone, intentionally focused practice that was used, in conjunction with specific targets, to monitor, control and sometimes punish particular aspects of employee behavior. The HR processes in productivity-oriented organizations were used to deepen the impact of HR practices rather than to extend their impact as was the case in the organizations with commitment-oriented HR philosophies. The use of these practices and processes produced employee behaviors that were in keeping with the HR philosophy, and its focus on high productivity and efficiency. While this approach resulted in low morale, frustration with a lack of training and career opportunities, low levels of job satisfaction, and the inability to use skills on the part of employees, it also resulted in high productivity because those who did not reach the required targets simply lost their jobs.

the important role that HR philosophy and HR processes play in the working of HR systems, the choices that firms have in the ways in which they configure their HR systems, and the outcomes that may result.

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ROLE OF LEADERSHIP POTENTIAL FOR MANAGERS

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ABSTRACT

Identifying leadership potential, determining the right development actions for them, and putting high potential individuals into key development roles quickly can differentiate successful organizations. Sometimes leadership potential means becoming more professionally competent in an area of the organization that the individual is already working in, sometimes it means taking on new responsibilities in another area of the organization, and sometimes it means taking on leadership responsibilities. What we do know is that Leadership potential, regardless of the direction, involves a combination of both personality and cognitive ability. The Leadership Potential Report (LPR) is a unique and powerful tool that assesses both of these aspects of Leadership: personality and cognitive ability.

The Personality Side of Leadership Potential

The Leadership Potential Report (LPR) is a tool that assesses the behaviors that relate to leadership potential for professional growth and career advancement. This includes demonstrating behaviors that relate to consistently growing the breadth of one's career-related capabilities; taking on higher-level responsibilities; showing motivation for advancing into future roles with increased responsibilities; and showing commitment to life-long growth and learning. There are four broad dimensions assessed (McCarthy, 2012):

- Interpersonal Versatility – an individual's style in taking charge and influencing groups.

- Thinking Versatility – an individual's style in analyzing and adapting to changing situations.
- Results Versatility – an individual's style in taking initiative and working to achieve goals.
- Emotional Versatility – an individual's style for handling stress and confidence.

The Cognitive Ability Side of Leadership Potential

The LPR assesses the reasoning skills related to career growth and professional advancement. This includes the degree to which the individual learns new concepts, solves problems, and spots trends in an accurate and efficient manner. Specifically, the LPR measures Verbal Reasoning, Numerical Reasoning, and Abstract Reasoning.

The LPR was developed specifically to identify high potentials and assist in succession planning. This makes it a powerful tool in the workplace for any Talent Management initiative as it can be used for issues related to:

- Selection – the LPR offers critical insights into candidates' potential and job fit, spotlighting strengths and potential weaknesses for their career advancement.
- Development – the LPR identifies the benefits and limitations of scores across the personality scales as well as providing specific development recommendations.
- Leadership – the LPR discusses a person's leadership style, offering detailed descriptions of when it may be useful and when it may not be effective.
- Succession Planning – the LPR provides a 9-Box Talent Management Worksheet to aid in identifying Top Talent and what to do to prepare people for future roles.

Validation studies have proven the effectiveness of the Leadership Potential Report:

1. Measures the potential for career advancement of applicants and employees in a highly reliable manner, especially when compared to other personality and cognitive ability tests.
2. Optimizes the fit between people and position requirements specific to each organization.
3. Reduces cost of assessing internal or external candidates for job opportunities and roles.
4. Helps assure optimal quality, productivity, and effectiveness of the workforce by aiding in the advancement of high potentials into higher levels of responsibility within the organization.

Assessing Potential

Identifying talent, determining the right development for each of them, and putting them into key development roles quickly differentiates successful organizations. Identifying internal talent is half the battle. When an organization does not have enough talent in-house, they have to know where and what they are missing and how to identify, recruit, and hire the people with the necessary talent. This is known as Talent Management (Timms, 2016).

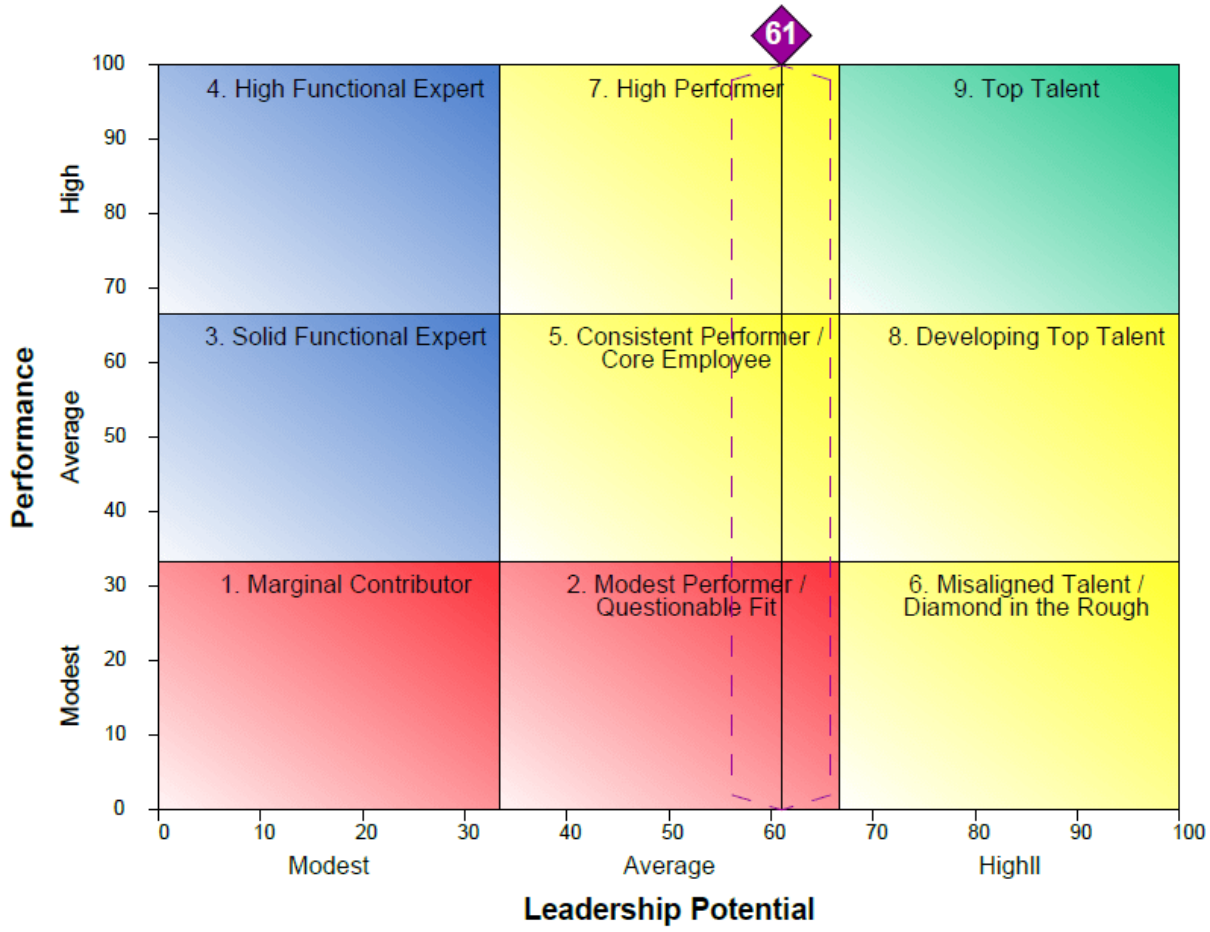
To identify talent, employees or applicants need to be assessed in some way. There are two components to measure to identify any given individual's talent:

1. Performance – an individual's effectiveness and results in his or her current role.
2. Potential – an individual's predicted future performance if given development opportunities and increased responsibility.

The 9 Box Talent Matrix

The most emphasis for assessment of Performance and Potential is during the selection, development, and succession planning processes. Using assessments helps determine who to

hire, what people need to develop, and who are the future leaders or the organization. To aid in these processes and in differentiating between employees on talent, 9-box grid is used.



Source:

(Nine-Box Matrix for Succession Planning and Development, n.d.)

Understanding where individuals lie on the 9-Box Grid allows organizations to set up specific development plans for each individual based on their combined potential and recent performance. This results in more specific and focused development for each individual, allowing the organization to put individuals in the critical or key opportunities that will help them develop to their full potential.

Unfortunately, many organizations struggle with effectively implementing processes to leverage tools like the 9-box grid. A common complaint within talent management is that individuals often seem to "appear" and "disappear" from their list of high talent within the organization – people who at one point were considered Developing Top Talent and potential future leaders might be found to be a Consistent Performer the next year. This is because many organizations struggle with a stable definition of potential and they put too much weight on performance when plotting people into the 9-boxes. It is then critical for organizations to understand what is meant by potential. (The GE-McKinsey Nine-Box Matrix, n.d.)

Leadership Development

Many leadership development programs focus on particular skills or situations which may or may not be relevant in the future or for each participant. Assessments are used to identify and focus on the characteristics specific to each individual that will improve their effectiveness and accelerate their career. From there, there are created actionable development plans around the high impact areas that will have the greatest effect on both the individual's and the organization's success.

- *Personality Assessment*

  This gives insights into the underlying drivers of how they may act or interact with others. It also gives an idea of an individual's preferred leadership style – how they tend to take charge and lead others.

- *Reasoning & Cognitive Ability Assessment*

  This gives insight into their ability to handle higher level roles and the increasing complexity of problems they may encounter.

- *360 Degree Assessment*

Direct reports, bosses, clients, peers, etc. are asked to rate the leader's effectiveness on specific behaviors related to each competency as well as providing self-ratings. This gives multiple perspectives of where a leader may be strong and where they may benefit from development.

- *Coaching*

Based on assessments and development feedback, tailored coaching is provided to leaders. Through a thorough analysis of each assessment, it is possible to create a specific individual development plan for the leader to improve their effectiveness and accelerate their career. Development recommendations are specific, step-by-step actionable activities that through regular practice, will improve a leader's ability or performance in the given area.

Talent Management Pipeline

Talent management is the anticipation of human capital needs for the organization and the planning to meet those needs. It is widely popular but can be a difficult process for an organization to use effectively. To implement talent management initiatives, organizations must understand how to source, attract, select, train, develop, retain, promote, and move employees through the organization. Doing so effectively improves business value and helps reach organizational goals by garnering more and building up current talent within the organization. What talent management becomes is the art of putting the right people in the right place and utilizing them to their full potential.

The organizations that thrive and do this effectively select and build their employees through their Talent Management Pipeline. The Talent Management Pipeline is the process, from start to finish, of identifying, hiring, and developing the people who will be successful in your organization and giving them the tools to continue to be successful. (Timms, 2016)

There are 4 Steps of the Pipeline:

1. Define the workplace competencies that drive success within the organization.
2. Assess applicants and employees to locate high potentials and future leaders for the company.
3. Select individuals to hire and promote who will continue to drive organizational success.
4. Develop the strengths and competencies of leaders and employees to produce a high-performing workforce.

Conclusion

Leadership development is a key facet of organizational success. Developing leaders and leadership teams can enhance productivity, teamwork, communication, and the bottom line. With the help of above-mentioned tools organizations can identify high potential leaders and implement strategies for enhancing leadership strengths.

By providing solutions all along the Talent Management Pipeline, it is ensured that organization is identifying the talent not only today, but the talent they may need for the future.

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Remark on Right Continuous Exponential Martingales

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Abstract Using $\langle M^c \rangle$ and jumps of a local martingale M, we characterize the event where the stochastic exponential $\mathcal{E}(M)$ equals to zero.

2000 Mathematics Subject Classification: 60 G44.

Keywords: Local martingale, Stochastic exponential with jumps.

1. Introduction. Let us introduce a basic probability space $\big(\Omega, \mathcal{F}, P\big)$ and a right continuous filtration $(\mathcal{F}_t)_{0\leq t<\infty}$ satisfying usual conditions. Let $\mathcal{F}_\infty$ be the smallest $\sigma-$Algebra containing all $\mathcal{F}_t$ for $t < \infty$ and let $M = (M_t)_{t\geq 0}$ be a local martingale on the stochastic interval $[[0; T[[$, where T is a stopping time. Denote by $\triangle M_t = M_t - M_{t-}$ jumps of M and by $\mathcal{E}(M)$ the stochastic exponential of the local martingale M:

$$\mathcal{E}_t(M) = \exp\big\{M_t - \frac{1}{2}\langle M^c\rangle_t\big\} \prod_{0<s\leq t} (1 + \triangle M_s)e^{-\triangle M_s},$$

where M^c denotes a continuous martingale part of M. It is well known that $\mathcal{E}_t(M) = 1 + \int_0^t \mathcal{E}_{s-}(M)dM_s$, so it is clear that for local martingale M the associated stochastic exponential $\mathcal{E}(M)$ is a local martingale. Throughout of this paper we assume that $\triangle M_t \geq -1$ which implies that $\mathcal{E}(M)$ is a non-negative local martingale and therefore supermartingale. In case when $\mathcal{E}(M)$ is a uniformly integrable martingale on $[[0;T]]$, we can define a new probability measure using Radon-Nikodym derivative: $dQ = \mathcal{E}_T(M)dP$. It is clear that $Q << P$ and if $P\{\mathcal{E}_T(M) > 0\} = 1$, then P and Q will be equivalent probability measures. To know whether $P \sim Q$ or not, we must study the set $\{\mathcal{E}_T(M) = 0\}$. In case when $M = M^c$ it was shown by Kazamaki [2] that $\{\mathcal{E}_T(M^c) = 0\} = \{\langle M^c \rangle_T = \infty\}$. For general M, in 1978, J. Jacod [1] proved that

$$\{\mathcal{E}_\infty(M) > 0\} = \Big\{\langle M^c \rangle_\infty + \Big(\sum_s \frac{(\triangle M_s)^2}{1 + |\triangle M_s|}\Big)^p_\infty + \int_0^\infty \frac{1}{\mathcal{E}_{s-}(M)} dB_s < \infty\Big\}$$

where $(\cdot)^p$ denotes predictable dual projection (compensator) and B_s is the predictable, non-decreasing process from the Doob-Meyer decomposition of $\mathcal{E}(M)$. In 2019 M. Larsson and J. Ruf [3] gave us the set inclusion

$$\{\lim_{t\uparrow\tau} \mathcal{E}_t(M) = 0\} \subset \big\{\lim_{t\uparrow\tau} M_t = -\infty\big\} \cup \{[M]_\tau = \infty\} \cup \{\triangle M_t = -1, t \in [0;\tau)\}$$

for any predictable stopping time τ. With this they proved, that if in addition $\triangle M \geq -1$ and $\overline{\lim}_{t\uparrow\tau} M_t < \infty$, then the reverse set inclusion also holds. The aim of this paper is to characterize the set $\{\mathcal{E}_T(M) = 0\}$ using $\langle M^c \rangle_T$ and $\sum_{s\leq T} \frac{(\triangle M_s)^2}{1+\triangle M_s}$:

Theorem 1 Let M be a local martingale with $\triangle M_s \geq -1$ and $E \triangle M_\sigma < \infty$, for any $\sigma < \infty$. Then the following set equality holds true P a. s.

$$\big\{\mathcal{E}_T(M) = 0\big\} = \Big\{\langle M^c \rangle_T + \sum_{s\leq T} \frac{(\triangle M_s)^2}{1 + \triangle M_s} = \infty\Big\}.$$

Remark 1 Notice, that we need the condition $E \triangle M_\sigma < \infty$ to prove the inclusion $\subset$, but for the reverse inclusion the above mentioned condition is not necessary.

Remark 2 In the contrary to the result from Jacod [1], we are not using the

additional increasing process B_t and the compensator, but we have restriction on jumps of M. In their result Larsson and Ruf [3] used the predictable stopping time τ and they have additional restriction on M to obtain the set equality. We also have additional but different kind of restriction on jumps on M and with this we use any kind of stopping times.

Proof of the Theorem 1: It is well known, that $M = M^c + M^d$ where M^c is a continuous martingale part of M, and M^d - purely discontinuous, which means that M^d is orthogonal to any continuous local martingale. It is easy to check that $\mathcal{E}_T(M) = \mathcal{E}_T(M^c)\mathcal{E}_T(M^d)$, so $\{\mathcal{E}_T(M) = 0\} = \{\mathcal{E}_T(M^c) = 0\} \cup \{\mathcal{E}_T(M^d) = 0\}$. It is well known that $\{\mathcal{E}_T(M^c) = 0\} = \{\langle M^c\rangle_T = \infty\}$ Kazamaki [2], so the set equality

$$\big\{\mathcal{E}_T(M^d) = 0\big\} = \big\{\sum_{s\le T} \frac{(\triangle M_s)^2}{1+\triangle M_s} = \infty\big\} \tag{1}$$

will complete the proof of Theorem 1, because

$$\Big\{\langle M^c\rangle_T + \sum_{s\le T} \frac{(\triangle M_s)^2}{1+\triangle M_s} = \infty\Big\} = \{\langle M^c\rangle_T = \infty\} \cup \Big\{\sum_{s\le T} \frac{(\triangle M_s)^2}{1+\triangle M_s} = \infty\Big\}.$$

If $\triangle M_s = -1$ for some $s \le T$, then it is obvious that $\mathcal{E}_T(M^d) = 0$ and $\sum_{s\le T} \frac{(\triangle M_s)^2}{1+\triangle M_s} = \infty$, so we left to show equality (1) when $\triangle M_s > -1$.

First let us show that $\big\{\sum_{s\le T} \frac{(\triangle M_s)^2}{1+\triangle M_s} = \infty\big\} \subset \big\{\mathcal{E}_T(M^d) = 0\big\}$. An easy calculations show that

$$\mathcal{E}_T^2\Big(\frac{1}{2}M^d\Big) = \exp\Big\{M_T^d + \sum_{s\le T}\big[2\ln\big(1+\frac{1}{2}\triangle M_s\big) - \triangle M_s\big]\Big\} =$$

$$\mathcal{E}_T(M^d)\exp\Big\{\sum_{s\le T}\ln\frac{(1+\frac{1}{2}\triangle M_s)^2}{1+\triangle M_s}\Big\}$$

and from this we obtain:

$$\mathcal{E}_T(M^d) = \mathcal{E}_T^2\Big(\frac{1}{2}M^d\Big)\exp\Big\{-\sum_{s\le T}\ln\Big(1+\frac{1}{4}\cdot\frac{(\triangle M_s)^2}{1+\triangle M_s}\Big)\Big\}.$$

The supermartingale property of $\mathcal{E}\big(\frac{1}{2}M^d\big)$ implies $P\{\mathcal{E}_T(\frac{1}{2}M^d) < \infty\} = 1$, so we obtain that $\big\{\sum_{s\le T}\ln\big(1+\frac{1}{4}\cdot\frac{(\triangle M_s)^2}{1+\triangle M_s}\big)=\infty\big\}\subset\{\mathcal{E}_T(M^d)=0\}$. Now we will have the following chain of set inclusion and equality:

$$\Big\{\sum_{s\le T}\frac{(\triangle M_s)^2}{1+\triangle M_s}=\infty\Big\}=\Big\{1+\frac{1}{4}\sum_{s\le T}\frac{(\triangle M_s)^2}{1+\triangle M_s}=\infty\Big\}\subset$$

$$\Big\{\prod_{s\le T}\Big(1+\frac{1}{4}\cdot\frac{(\triangle M_s)^2}{1+\triangle M_s}\Big)=\infty\Big\}=\Big\{e^{\sum_{s\le T}\ln\left(1+\frac{1}{4}\cdot\frac{(\triangle M_s)^2}{1+\triangle M_s}\right)}=\infty\Big\}=$$

$$\Big\{\sum_{s\le T}\ln\Big(1+\frac{1}{4}\cdot\frac{(\triangle M_s)^2}{1+\triangle M_s}\Big)=\infty\Big\}\subset\{\mathcal{E}_T(M^d)=0\}.$$

Now it is time to prove the reverse set inclusion: $\{\mathcal{E}_T(M^d)=0\}\subset\Big\{\sum_{s\le T}\frac{(\triangle M_s)^2}{1+\triangle M_s}=\infty\Big\}$. Let $\mu(\omega,t,x)$ be the jump counting measure for local martingale M and $\nu(\omega,t,x)$ be its compensator. Then we have the representation: $M_t^d=\int_0^t\int_{[-1;+\infty)}xd(\mu-\nu)=\sum_{s\le t}\triangle M_s-\int_0^t\int_{[-1;+\infty)}xd\nu$. Define the local martingales

$$M_t^1=\int_0^t\int_{[1;+\infty)}xd(\mu-\nu);\qquad M_t^2=\int_0^t\int_{(-1;1)}xd(\mu-\nu).$$

It is clear that $\triangle M_t^1\ge 1$ and $|\triangle M_t^2|<1$. With this it is easy to check that $M_t^d=M_t^1+M_t^2$, $\mathcal{E}_T(M^d)=\mathcal{E}_T(M^1)\mathcal{E}_T(M^2)$ and consequently $\{\mathcal{E}_T(M^d)=0\}=\{\mathcal{E}_T(M^1)=0\}\cup\{\mathcal{E}_T(M^2)=0\}$.

I. In this part we prove inclusion $\{\mathcal{E}_T(M^1)=0\}\subset\big\{\sum_{s\le T}\frac{(\triangle M_s^1)^2}{1+\triangle M_s^1}=\infty\big\}$. It is clear that $\mathcal{E}_T(M^1)=\exp\big\{-\int_0^T\int_{[1;+\infty)}xd\nu+\sum_{s\le T}\ln(1+\triangle M_s^1)\big\}$ and the inequality $\triangle M_s^1\ge 1$ implies $\{\mathcal{E}_T(M^1)=0\}\subset\big\{\int_0^T\int_{[1;+\infty)}xd\nu=\infty\big\}$. This and the condition $E\triangle M_\sigma<\infty$, for any $\sigma<\infty$, gives us possibility to use Theorem 2.6.1 from [4] to obtain the set equality:

$$\Big\{\int_0^T\int_{[1;+\infty)}^x d\nu=\infty\Big\}=\Big\{\int_0^T\int_{[1;+\infty)}xd\mu=\infty\Big\}=\Big\{\sum_{s\le T}\triangle M_s^1=\infty\Big\}.$$

Inequality $\triangle M_s^1 \geq 1$ implies that $\triangle M_s^1 \leq \frac{2(\triangle M_s^1)^2}{1+\triangle M_s^1}$, so using this we obtain:

$$\{\mathcal{E}_T(M^1) = 0\} \subset \Big\{ \sum_{s\leq T} \frac{2(\triangle M_s^1)^2}{1+\triangle M_s^1} = \infty \Big\} = \Big\{ \sum_{s\leq T} \frac{(\triangle M_s^1)^2}{1+\triangle M_s^1} = \infty \Big\}.$$

II. Now we will show that $\{\mathcal{E}_T(M^2) = 0\} \subset \big\{ \sum_{s\leq T} \frac{(\triangle M_s^2)^2}{1+\triangle M_s^2} = \infty \big\}$.

$$\mathcal{E}_T(M^2)\mathcal{E}_T^2\big(-\frac{1}{2}M^2\big) = \exp\Big\{ M_T^2 + \sum_{s\leq T} \big[\ln(1+\triangle M_s^2) - \triangle M_s^2\big] - M_T^2 +$$

$$\sum_{s\leq T} \big[2\ln(1-\frac{1}{2}\triangle M_s^2) + \triangle M_s^2\big]\Big\} = \exp\Big\{ \sum_{s\leq T} \ln(1+\triangle M_s^2)\big(1-\frac{1}{2}\triangle M_s^2\big)^2\Big\}.$$

From the last equality and the supermartingale property of $\mathcal{E}\big(-\frac{1}{2}M^d\big)$ we deduce that

$$\big\{\mathcal{E}_T(M^2) = 0\big\} \subset \Big\{ -\sum_{s\leq T} \ln(1+\triangle M_s^2)\big(1-\frac{1}{2}\triangle M_s^2\big)^2 = \infty \Big\}. \qquad (2)$$

Using $|\triangle M_s^2| < 1$ and Lemma 1 from Appendix we obtain $-\ln(1+\triangle M_s^2)\big(1-\frac{1}{2}\triangle M_s^2\big)^2 \leq \frac{2(\triangle M_s^2)^2}{1+\triangle M_s^2}$ and this with (1) gives us inclusion:

$$\{\mathcal{E}_T(M^2) = 0\} \subset \Big\{ \sum_{s\leq T} \frac{2(\triangle M_s^2)^2}{1+\triangle M_s^2} = \infty \Big\} = \Big\{ \sum_{s\leq T} \frac{(\triangle M_s^2)^2}{1+\triangle M_s^2} = \infty \Big\}.$$

Now if we summarize results of I and II, we will have:

$$\{\mathcal{E}_T(M^d) = 0\} = \{\mathcal{E}_T(M^1) = 0\} \cup \{\mathcal{E}_T(M^2) = 0\} \subset$$

$$\Big\{ \sum_{s\leq T} \frac{(\triangle M_s^1)^2}{1+\triangle M_s^1} = \infty \Big\} \cup \Big\{ \sum_{s\leq T} \frac{(\triangle M_s^2)^2}{1+\triangle M_s^2} = \infty \Big\} =$$

$$\Big\{ \sum_{s\leq T} \frac{(\triangle M_s^1)^2}{1+\triangle M_s^1} + \sum_{s\leq T} \frac{(\triangle M_s^2)^2}{1+\triangle M_s^2} = \infty \Big\} = \Big\{ \sum_{s\leq T} \frac{(\triangle M_s)^2}{1+\triangle M_s} = \infty \Big\}.$$

Finally we obtain $\{\mathcal{E}_T(M^d)=0\}=\Big\{\sum_{s\le T}\frac{(\triangle M_s)^2}{1+\triangle M_s}=\infty\Big\}$ and

$$\{\mathcal{E}_T(M)=0\}=\{\mathcal{E}_T(M^c)=0\}\cup\{\mathcal{E}_T(M^d)=0\}=$$

$$\{\langle M^c\rangle_T=\infty\}\cup\Big\{\sum_{s\le T}\frac{(\triangle M_s)^2}{1+\triangle M_s}=\infty\Big\}=\Big\{\langle M^c\rangle_T+\sum_{s\le T}\frac{(\triangle M_s)^2}{1+\triangle M_s}=\infty\Big\}.$$

□

4. Appendix.

Lemma 1. $k(x)=\frac{2x^2}{1+x}+\ln(1+x)(1-\frac{1}{2}x)^2\ge 0$ for any $x\in(-1;1)$.
Proof.

$$k'(x)=\frac{4x(1+x)-2x^2}{(1+x)^2}+\frac{(1-\frac{1}{2}x)^2-(1+x)(1-\frac{1}{2}x)}{(1+x)(1-\frac{1}{2}x)^2}=$$

$$\frac{2x^2+4x}{(1+x)^2}-\frac{3x}{(1+x)(2-x)}=\frac{-2x^3-3x^2+5x}{(1+x)^2(2-x)}=\frac{x(2x+5)(1-x)}{(1+x)^2(2-x)}.$$

It is obvious that $k'(0)=0$, $k'(x)<0$ when $x\in(-1;0)$ and $k'(x)>0$ when $x\in(0;1)$. So $x=0$ is a minimum point and because $k(0)=0$, we can deduce that $k(x)\ge 0$ for $x\in(-1;1)$. □

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Role of Human Capital in Contemporary Organizations

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Abstract

This paper explains the meaning, importance and gives reasons why company should create and take care of human capital. The goal of this paper is to explain some metrics of human capital, its efficiency, show practical examples point out the possible implementation of knowledge in this area in teaching the economic subjects.

This paper shows how human capital can increase compatibility of the company in the market, increase its value and make company business more profitable.

What is Human Capital?

For all types of organizations, being of human capital is crucial. First of all it is important to understand what does human capital mean. In general, human capital is described as the skills, training, and health acquired through on the job training and education. Human capital may also be defined as the "endowment of abilities to produce that exists in each human being. “

The idea of human capital originates from earlier 18-th century and belongs to Adam Smith. An economist and Nobel Laureate from the University of Chicago Gary Becker introduced and popularized modern theories, Jacob Mincer, and Theodore Schultz. Paul Romer was awarded for his work using Human Capital as a key factor in 2018 with Nobel Prize for Economics who founded the modern innovation-driven approach to understanding economic growth.

The term of “human capital” is a key element to improve the assets of the organization. It can be seen as sustainable competitive advantage and a factor of increased employee efficiency. Some theorists apply rules to prove the ability to create useful competitions between companies by developing individual human resources.

Recently, managers of organizations have found that human resources to have greatest importance in gaining sustainable competitive advantage and efficiency. In the world where education, along with knowledge and communication with customers have gained increasing importance, human capital, that is the bucket of knowledge, technical skills, creativity, and experience of the organization, gains great importance, thus labour force is the most productive and also highly valued and not costly assets.

To form strong human capital it is important to start investing in education and health. Investing in health is crucial as it increases efficiency of labor, decreases sickness and finally aids development of economics.

Though the act of researching, assessing and controlling human resources is more difficult than other operations of the organization, since the rarest and most complicated resources in knowledge-based economy are human resources. Mostly top management concentrates on their strategies on tangible and obvious factors of the organization such as technology and using physical and financial resources.

Why Company Needs to Have Human Capital?

Due to today’s characteristics of global economy contemporary technologies in producing various and new products, cannot create competitive advantage for the organizations by itself. For this reason, organizations must concentrate on other factors such as human intellectual capital in order to gain competitive advantage and improve their survival. That’s why it ncreases needs of organizations to effectively attract, develop, and preserve talents.

According to the above mentioned, today, human capital has gained importance more than any other tangible resources for the organization. In this paper is explained the concept of human capital, its importance, and the characteristics of human capital.

Human capital simply is physical capital just like properties, equipment and financial capital. In the past ages, the share of physical capital in GDP in the economy of advanced countries has sharply fallen, while the share of human capital has risen. This rise in the share of human capital in GDP has created the other level concept of knowledge economy. Other types of capital are considered as inputs, of the process of producing goods and services. But human capital is not considered as a simple input, as it plays a more complicated role in the process of producing goods or providing services.

In a new definition of human capital, it is considered as a collection of features, life trade, knowledge, creativity, innovation, and energy, which people invest it in their work. The fact that employees are the most valuable resources within the organization is not new. Empowering labor force and giving them opportunity to make decision will increase their motivation and reduce their resistance to organizational changes.

Management of workforce knowledge is an effective field, that is created by various factors such as human resources, organizational development, changes in management and management system, information technology, management credit and estimating performance. Knowledge management is the process of having special kind of knowledge and using it in order to stimulate process of development and innovation. Human resources can help the company with having competitive advantage and operating comprehensive quality plans. Employees can create the predictions at different level of the organization, define the values, missions and goals, design strategic plans, and implement those plans according to values. Value added can be reinforced through motivating and training the employees.

How Company Should Increase Human Capital?

Five basic steps help company increase human capital:

1. Constant trainings;
2. Monitoring Performance;
3. Direct Communication;
4. Defined Job Responsibilities;
5. Motivation.

It is right way to say that the success and failure of any organization depends on its employees. Every employee in its very own way contributes the development of the organization. One of the major responsibilities of a HRM is to hire and distribute the right talent for the organization in short to recruit the right person for the right role. Job mismatch may lead to confusions and eventually decrease the productivity of the company and output. Distributing all the new employees to their job places must not be neglected. Documents and employee handbooks are not sufficient to welcome new employee to its job place. HR managers should design a suitable introduction program that will not only acquaint a new individual with the system and policies but also makes feel comfortable within the system.

It is essential for every organization to upgrade the skills of its employees to cope well with the changing times. Employees must be trained from time to time, to be able to utilize their knowledge to the fullest when required and become one of the key resources for their organization. Training managers and HR managers must not design training programs just for the sake of it. They have to estimate how training program will benefit the employees. Management can also sponsor the education of their employees that will sharpen their skills and increase human capital. Necessary, sharply tailored and realistic training programs increase efficiency of an employee, who further increases the productivity of organization.

Monitoring performance can also increase human capital. To understand what your employees are up to, managers must take regular feedbacks from their subordinates. Introducing the concept of weekly report system (that can be oral, written or online). At the end of every week, employees reporting to their bosses will be evaluated according to their performances and given comments and feedbacks. This way managers can also keep a track on their employees' progress.

Employees should be able to walk up to their managers in case of any queries. The dictator approach is not effective and does not work nowadays. Problems have to be discussed and suggestions should be given along with feedbacks and ideas. Never should be ignored even the tiniest problem. Any problem left without attention may lead to a serious problem later on. This way employees feel attached towards the organization. Nobody ever knows when someone might come up with a wonderful idea.

All the job responsibilities and expectations of employees must be clearly defined to expect any misunderstanding. Key responsibilities should be designed by individual's past work experience, education and areas of interest.

Employees has to feel motivated from time to time according to feedbacks from work records. And when working harder should be motivation for others and draw inspiration from them. Good employee talents and job done must be recognized either by offering them moral incentives or giving them a decent salary hike.

When Should Company Start Increasing Human Capital?

At every stage of the business development, company must strive on increasing human capital. Managers should have desire to develop and retain human capital or understand the need for management of talent at their company.

The general demand for top-notch employees and outstanding managers is to meet the corporation 's objectives and enhance its business performance. For this reason, many organizations focus on business strategy and means to achieve growth, few think about the characteristics of the human capital required to realize their strategy and growth objectives.

How to prioritize investment in human resources and to build a differential strategy based on the various employee groups found in the company? It can be answered the following - e human capital strategy process, that not only defines the role and profile of the managers themselves but also explains and gives instructions how managers should handle employee development and prepare staff for their duties. It governs HRM over the entire employee life

cycle – from specifying which employees need to be recruited, to subsequent training, career path management in the organization and, ultimately, retirement. This strategy significantly affects the organization as an employer, to other interested parties, such as future candidates, competitors and customers. The first real step in the process is defining what to focus on, the second step is defining what to focus on an employee level and the third and final step in the process is selection.

The several examples of the companies that are highly innovative due to their strong human capital, are the following: FedEx Corporation, Sainsbury's, Marks & Spencer, Rolls-Royce, Shell, Ford, LinkedIN, Twitter, Nissan, Google etc.

Summary

In summary, human capital has the most important role in people's development, improving the life and income, increasing knowledge, skill, and product capacities, also increasing economic growth and reducing poverty. In this constantly changing world, where there are new revolutions in fighting against capitalism, human forces gain more and more importance than ever. Future researches of human capital will have two goals - measure the gaps within human capital and how human capital can lead to more efficiency and revenue. Obviously employees and individuals apply human capital to their job, they receive their investment reward from human capital through salary, benefits, job satisfaction, and more learning opportunities as well as promotion. These achievements allow families and governments to invest a large part of their resources in education, health and training human force. It is worth mentioning that economic attitude towards human capital (education, health and training labor force) is vital for country's efficiency and economic success.

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Modeling medical procedure protocols adoption priorities in healthcare

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Abstract

Protocols and procedures are one of the major requirements for quality and timely medical services. In addition, they can significantly reduce treatment time what determines the medical service cost. Adoption of medical service protocols are difficult and entails significant human and material resource requirements. In the article we are considering mathematical and computer model of the protocols adoption priorities and presenting numerical solution using real data from of the clinics. The medical services protocol adoption priorities results received from multi-objective optimization of service time are further valuated by probability and impact of such medical conditions.

Introduction

Protocols can be defined as set of medical guidelines and instructions used in different medical situations. In addition to sequence of recommended actions of medical services protocols which can be freely followed by medical personal, some of them can require authorization. Protocols adoption ensures that patients are receiving quality and timely medical services.

Adoption of the medical protocols are recommended by several healthcare organizations (e.g., WHO [1], Council of Europe [2]) and are based on best practices revealed from medical guidelines used in different clinics. These studies had been conducted by numerous groups of researchers and organizations. Just to name few of them: [3], [4], [5], etc.

Adoption of the protocols requires additional material, financial, and human resources, and difficult to implement simultaneously for all medical conditions. Therefore, it's important to prioritize the medical conditions which can cause more harm to patent and can occur frequently. Also, should be selected the criteria which can measure the impact of different medical conditions on patents. For this purpose, in our model, we have chosen duration of medical services as measurements of effectiveness of actions in different medical conditions. In addition, duration of the medical services directly related to the total medical cost which should be paid by patents or healthcare insurance institutions.

For mathematical and computerized model calculations we have used the data for one year of one of the private clinics. Data have been chosen for 2018 year, before COVID-19 global pandemic in order to get pure information about medical cases without procedures enforced by virus dissemination.

Model description

Data contains different medical conditions and duration of procedures required for these conditions. We have classified the medical conditions and represented the duration as a percent increase of decrease from the mean duration of the classes of medical conditions and its required procedures. So, we get lognormally distributed duration differences in percentage. Also, we have removed all classes of medical conditions with zero standard deviation and records with zero or single cases in the class, which potentially can cause problems during model calculations. Finally, after data cleansing remained 5868 records and 133 different classes of medical conditions.

We have denoted by r - percent difference from mean of duration of medical services required for each k class of medical conditions, and by σ – standard deviation of duration.

For adoption priority calculations we are using Pareto multi-objective optimization method. We are revealing classes of medical conditions with highest duration and standard deviation. For this purpose, we are writing maximization problem of the following function:

$$\max f(x) = \left(\mu_1 R(x), \mu_2 V(x)\right) \tag{1}$$

where

$$R = \sum_{k=1}^{N} r_k x_k \tag{2}$$

and

$$V = \sum_{k=1}^{N} x_k^2 \sigma_k^2 \tag{3}$$

Substituting equations (2) and (3) into (1) we get

$$f(x) = \mu_1 \sum_{k=1}^{N} r_k x_k + \mu_2 \sum_{k=1}^{N} x_k^2 \sigma_k^2 \tag{4}$$

where

$\mu_1 = 1, \mu_2 = {}^1/_2$

After finding the partial derivatives from equation (4) we have the following:

$$\begin{aligned} \frac{\delta f(x)}{\delta x_1} &= r_1 + x_1 \sigma_1^2 \\ &\vdots \\ \frac{\delta f(x)}{\delta x_k} &= r_k + x_k \sigma_k^2 \end{aligned} \tag{5}$$

where

$k = \{1,2, \dots, N\}$

We are receiving system of equations which can be solved for x.

$$\begin{aligned} x_1\sigma_1^2 &= -r_1 \\ &\vdots \\ x_k\sigma_k^2 &= -r_k \end{aligned} \tag{6}$$

We can write in matrix form

$$X = -\Sigma^{-1}R \tag{7}$$

where

$\Sigma = \mathrm{diag}(\sigma_1^2, \dots, \sigma_N^2)$ – diagonal matrix of variances for each medical conditions' classes

$\mathrm{R} = (r_1, \dots, r_N)$ – column vector of mean percent differences for each medical conditions' classes

$\mathrm{X} = (x_1, \dots, x_N)$ – column vector of weights for each medical conditions' classes

In order normalize weights vectors we can use logistical function and find the impact of each class

$$c_k = \frac{1}{1 + e^{-x_k}} \tag{8}$$

Substituting (7) into (8) we will have in matrix form:

$$C = \frac{1}{1 + e^{\Sigma^{-1}R}} \tag{9}$$

Also, we can easily calculate the frequency of each class of medical conditions occurrence by the following way:

$$p_k = \frac{frequency_k}{\max\left(frequency_k\right)} \tag{10}$$

where

$c_k - concequencies\ of\ medical\ condition\ duration\ in\ [0,1]\ interval$

$p_k - frequency\ of\ medical\ condition\ in\ [0,1]\ interval$

Finally, we can calculate Priority Factor (priority of protocol adoption) for each class of medical conditions by the following way:

$$F_k = p_k + c_k - p_k c_k \qquad (11)$$

Based on priority factors we can establish ranking system of protocol adoption for different class of medical conditions.

Conclusion

In this model we are presenting numerical objective ranking system for each class of medical conditions and related protocols of medical services. Part of the ranking system results of this model calculated with given data are in the table below:

Class (Code)	Priority Factor	Ranking	Priority
000	0.6806	15	Medium
100	0.6936	13	Medium
110	0.8249	2	High
111	0.7914	4	High
161	0.5948	78	Medium
300	0.6209	38	Medium
AQ-	0.7154	10	High
ARB	0.5626	117	Medium
ATK	0.6960	12	Medium
B08	0.5630	112	Medium
C15	0.5773	104	Medium
C16	0.6263	34	Medium
C17	0.5809	98	Medium
C18	0.5961	75	Medium

In the table Priority are assigned based on the Priority Factor values by the following way: High – more than 0.7, Medium – between 0.3 and 0.7, and Low - less than 0.3.
Ranking system can reveal the priority of medical conditions for which should be adopted medical services protocols in order to allocated resources with high efficiency.

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Building High Performance Workplace in Georgian Organizations

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ABSTRACT

Many organizations require collaboration to achieve projects or organizational goals. Teamwork can help solve difficult problems and improve productivity. High-performance teams in the workplace make this success possible by combining individual talent to achieve a common goal. Since the 1980s, organizations and companies placed an increasing number of important use teams in the workplace. Creating a highly functional team means more than just handing out tasks to a random group of people; This requires prior understanding and an effective approach.

One of the eternal questions of leaders is: What makes a High-Performance organization? Numerous companies claim to be high quality organizations. It is actually rare but instantly recognizable.

High quality organizations are role models for the organizational world. They represent real versions of the modern managerial ideal: an organization that excels in these many areas that it consistently outperforms most of its competitors over a long period of time.

Leading a high performance organization teaches and assists employees to achieve better results by supporting, assisting them, protecting them from outside interference, and making them accessible. Management holds people accountable for results, and it is crucial to always focus on the unwanted performer to achieve results, maintain clear accountability for performance, and make tough decisions. High-performance organization managers develop an effective, confident and rigorous management style by communicating values and making sure that the strategy is known and accepted by all members of the organization.

This paper explains the meaning, why it is necessary to create a highly performance workplace, what benefits all this brings to the company and how we can build all this in Georgian companies.

What is high performance workplace

To explain what a highly performance workplace is - it is a physical or virtual environment designed to make employees as efficient as possible in achieving business goals and values. A highly performance workplace is the result of an ongoing process for the company to enhance employee training, coaching, teamwork, and innovation. All of this is needed to create an effective work environment related to the company's financial income.

High performance workplaces come in a variety of shapes and sizes - from small and medium sized enterprises to large multinational organizations - But there are some common features, even if used for commercial success. This will contribute to a high level of staffing, good human resource practice and good practice of rewards and commitments. This is demonstrated by good management skills, commitment to flexible work policies, innovative reward schemes, employee information and consultation, as well as the organization's focus on skills development, as well as diversity knowledge and a good gender balance organization.

A high-performance culture is a set of behaviors and norms that leads an organization to achieve superior results. In other words, it is a culture led by a highly qualified organization that, according to the Cornell ILR School, is a company that achieves better financial and non-financial results (such as customer satisfaction, employee retention, etc.) than its peers over a long period of time. As mentioned above, a company culture is how an organization operates. When these behaviors and norms are aligned with organizational goals, customer needs, and employee priorities, the company is willing to achieve both financial results and other benefits such as employee retention and engagement.

Why is important high performance workplace

Business owners need employees who can do the job assigned to them, because the activities and behaviors of each employee determine the success of the company. It is a highly qualified environment that makes employees more trusting of each other, more independent in some matters and make their own decisions, and satisfied with their work. All this is necessary for employees to be more responsible, to work more productively, which is essential for the company, because these factors play an important role in the success of a business. High performance workplace have become very important as a source of competitive advantage in today's competitive business environment. Human resource management capabilities are important for attracting, selecting, retaining, motivating and developing the workforce in the organization. Meanwhile, organizational culture, which is viewed as organizational capital, can also be a driver of sustainable competitive advantage. Organizational values, as a reflection of the culture of the organization, asserts strategic issues such as strategic change, management decision-making, as well as creating employee responsibilities and the organization's interaction with external stakeholders. Organizations compete on the basis of their resources and capabilities. These dynamic and infinite mechanisms help the organizations reach, develop, and distribute their resources - Competitive advantage over other firms was maintained. Resource-based view, by setting people or firm people. First of all, resources, among all possibilities, imply that human resource management systems contribute to systems organizational success through firm-specific empowerment.

3 most important things that make a company a good place to work

Probably a place that inspires and motivates the people inside it to do everything, right? This magical environment, or the success that follows it, just is not visible; You need to make a commitment to make your business the best place to work.

- Excellent communication between management and staff.

Make sure you contact your employees regularly and make sure your methods are not boring, especially if you have senior staff. Communication ideas include regular meetings without holding weekly meetings, the same conversations with your employees so often, asking employees if they would like to receive updates via email or SMS.

Also be sure to contact them when there is news, such as launching new offers or entering new big customers. Making sure your employees are regularly updated is not only one way to improve communication, but also increases morale and engagement.

- Freedom of learning and professional growth for employees

These processes of identity transformation are particularly important in modern labor markets, where more flexible organizations feel less responsible for supporting their employees' career paths and where, at the same time, people need to learn more intensively and more often. Individuals' commitment to learning in the workplace lies both in the transformation of their personality and in their work. In our view, this process of transformational workplace learning should be implemented by organizations to foster a growing need for flexibility in work and to promote a model of autonomous and adaptive workers.

- Family environment among group members

Team members who cannot trust one other or who do not believe in the process and goals of the team seldom find success. Effective teams focus on solving problems. Trust is an adjunct

of effective communication; there can be trust between team members only if they are allowed to air their views freely. This is the reason why organizations often undertake team-building exercises that put team members in positions of trust.

Little overview about performance workplace in Georgian organizations

Today, unfortunately, most Georgian companies do not consider it necessary to develop a highly performance workplace. They have staff and they think that these staff no longer need development, retraining, training. Their main problem is that paying for all this is a pointless expense, but they do not realize that the cost now will bring more profit and success to the company in the future.

If I take examples from my personal work experience, the companies where I worked nowhere paid attention to the above issue. According to the top management, the employees were sufficiently skilled in their specialty and performed their job in the best possible way, at least satisfactorily. Therefore, retraining of employed staff was less of a priority for management. As for the existing service, even though the shareholder of the company is a European company, the above issue still ends improperly. Staff training is not systematic, but once a year despite the sacrifices.

Conclusion

A high-performance workplace is a system in which an organization is maximally aligned with their social system (people and their interactions) and technical system (equipment and processes). High-performance work requires large investments, including training, more selective hiring, higher salaries, and so on.

The work environment is one of the most important part of a company's success. It is directly related to the effective work of employees, more productivity and responsibility.

All this is necessary to achieve an effective working environment, which is related to the success of the company and of course the financial income.

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Utility maximization problem with uncertainty of success probability

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Abstract

Abstract. We study a robust portfolio optimization problem under model uncertainty for an investor with logarithmic or power utility.

1 Problems formulation

Let S_n be stock price defined by

$$S_{n+1}(\omega_1...\omega_n\omega_{n+1}) = (1_{(\omega_{n+1}=H)}u + 1_{(\omega_{n+1}=T)}d)S_n(\omega_1...\omega_n),$$

$\omega = \omega_1...\omega_N \in \{H,T\}^N,\ \tilde{q} = \frac{u-1-r}{u-d},\ \tilde{p} = \frac{1+r-d}{u-d}$. r is interest rate. Evidently that $S_N = S_0 u^{\sharp H(\omega)} d^{\sharp T(\omega)} = S_0 d^N (\frac{u}{d})^{\sharp H(\omega)}$. Let $\tilde{P}(\omega) = \tilde{p}^{\sharp H(\omega)}(1-\tilde{p})^{\sharp T(\omega)}$ be risk-neutral probability measure on $\{H,T\}^N$ and $P_p(\omega) = p^{\sharp H(\omega)}(1-p)^{\sharp T(\omega)}$ is reference measure. For

$$Z(p) = \frac{P}{\tilde{P}} = \left(\frac{p}{\tilde{p}}\right)^{\sharp H(\omega)} \left(\frac{1-p}{1-\tilde{p}}\right)^{\sharp T(\omega)} = \left(\frac{p}{\tilde{p}}\right)^{\sharp H} \left(\frac{q}{\tilde{q}}\right)^{\sharp T} = \left(\frac{p\tilde{q}}{\tilde{p}q}\right)^{\sharp H} \left(\frac{q}{\tilde{q}}\right)^N$$

we have

$$Z(p) = g\left(\frac{S_N}{S_0}\right),$$

where $g(x) = \left(\frac{q}{\tilde{q}d^{\log_{u/d}(p/q)}}\right)^N x^{\log_{u/d}(p/q)}$.

It is well known that utility maximization problem

$$\max_{\Delta} EU(X_N),$$

for wealth process $X_{n+1} = \Delta_n S_{n+1} + (1+r)(X_n - \Delta_n S_n)$ is equivalent to

$$\max_{V:\tilde{E}V=X_0'} EU(V),\ X_0' = X_0(1+r)^N.$$

The solution of utility maximization problem

$$\max_{V=X_0'} EZ(p)U(V)$$

when p is known exactly and U is either the logarithmic utility $U(x)=\ln(x)$ or a power utility $Ux)=\frac{1}{\gamma}x^{\gamma},\ \gamma<1,\ \gamma\neq 0$ is expressed explicitly and equal to

$$V=X_0'\frac{Z(p)^{\frac{1}{1-\gamma}}}{\tilde{E}Z(p)^{\frac{1}{1-\gamma}}},\ \gamma<1.$$

Our aim is to investigate the problem

$$\max_{V:\tilde{E}V=(1+r)^N X_0}\min_{p\in\Theta}\tilde{E}Z(p)U(V), \tag{1}$$

$$where\ Z(p)=\left(\frac{p}{\tilde{p}}\right)^{\sharp H(\omega)}\left(\frac{1-p}{1-\tilde{p}}\right)^{\sharp T(\omega)},\ \Theta\subset[0,1].$$

Finally the optimal $\hat{V}$ has to hedged, i,e. it will exists $X_0,X_1,...,X_N$ such that $X_N=\hat{V}$.

The general cases of Θ we consider are $\{p_0,p_1\},\ [p_0,p_1],\ [p_0,p_1]\backslash(\tilde{p}-\varepsilon,\tilde{p}+\varepsilon),\ p_0<p_1$. In the sequel we use the notation $\mathcal{P}(\Theta)$ for set of probability measures on Θ and $Z(\nu)=\int_\Theta Z(p)\nu(dp)$. Also we use the notation $Z^\alpha=\alpha Z(p_0)+(1-\alpha)Z(p_1),\alpha\in[0,1]$ for the $Z(\nu),\nu=\alpha\delta_{p_0}+(1-\alpha)\delta_{p_1}$. Since

$$\min_{p\in\Theta}\tilde{E}Z(p)U(V)=\min_{\nu\in\mathcal{P}(\Theta)}\tilde{E}Z(\nu)U(V),$$

the problem (1) we can change by

$$\min_{\nu\in\mathcal{P}(\Theta)}\max_{V:\tilde{E}V=(1+r)^N X_0}\tilde{E}Z(\nu)U(V), \tag{2}$$

which allows us to study at first

$$\max_{V:\tilde{E}V=(1+r)^N X_0}\tilde{E}Z(\nu)U(V).$$

$$\frac{1}{\gamma}X_0'^{\gamma}\tilde{E}Z(\nu)\frac{Z(\nu)^{\frac{\gamma}{1-\gamma}}}{\tilde{E}^{\gamma}Z(\nu)^{\frac{1}{1-\gamma}}}=\frac{1}{\gamma}X_0'^{\gamma}\tilde{E}^{1-\gamma}Z(\nu)^{\frac{1}{1-\gamma}},\ \gamma<1.$$

Hence it remains to solve

$$\min_{\nu}\tilde{E}^{1-\gamma}Z(\nu)^{\frac{1}{1-\gamma}},\ \gamma\neq 0$$

or

$$\min_{\nu}\tilde{E}Z(\nu)\ln Z(\nu),\ \gamma=0$$

2 The uncertainty of success probability

Let $\Theta = \{p_0, p_1\}$ and $\nu = \alpha\delta_{p_0} + (1-\alpha)\delta_{p_1}$. It needs to find $\min_{0\le\alpha\le1} f(\alpha)$, where

$$f(\alpha) = \begin{cases} \tilde{E}^{1-\gamma} Z(\alpha)^{\frac{1}{1-\gamma}}, \ \gamma \neq 0 \\ \tilde{E} Z(\alpha) \ln Z(\alpha), \ \gamma = 0. \end{cases}$$

Proposition 1. Let α^* be a minimum point of function $f(\alpha) = \tilde{E} Z^\alpha \ln Z^\alpha = E^\alpha \ln Z^\alpha$, *i.e.* $f(\alpha^*) = \min f(\alpha)$. Then

$$\alpha^* = \begin{cases} 1, \ if \ p_0 < p_1 \le \tilde{p}, \\ f'^{-1}(0) \text{ (i.e. the unique solution of } f'(\alpha^*) = 0), \ if \ p_0 < \tilde{p} < p_1, \\ 0, \ if \ \tilde{p} \le p_0 < p_1. \end{cases}$$

Moreover

$$\begin{aligned} f'(0) < 0, \ f'(1) > 0, \ if \ p_0 < \tilde{p} < p_1, \\ f'(\alpha) \ge 0, \ \alpha \in [0,1], \ if \ p_0 < p_1 \le \tilde{p}, \\ f'(\alpha) \le 0, \ \alpha \in [0,1], \ if \ \tilde{p} \le p_0 < p_1. \end{aligned}$$

Proof. Assume $p_0 < \tilde{p} < p_1$.

$$\begin{aligned} f'(\alpha) &= \tilde{E}(Z(p_0) - Z(p_1)) \ln Z^\alpha \\ &= \sum_{\omega\in\Omega} (p_0^{\sharp H(\omega)} q_0^{\sharp T(\omega)} - p_1^{\sharp H(\omega)} q_1^{\sharp T(\omega)}) \ln \left(\frac{\alpha p_0^{\sharp H(\omega)} q_0^{\sharp T(\omega)} + (1-\alpha) p_1^{\sharp H(\omega)} q_1^{\sharp T(\omega)}}{\tilde{p}^{\sharp H(\omega)} \tilde{q}^{\sharp T(\omega)}} \right) \\ = \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln \left(\alpha \left(\frac{p_0}{\tilde{p}}\right)^k \left(\frac{q_0}{\tilde{q}}\right)^{N-k} + (1-\alpha) \left(\frac{p_1}{\tilde{p}}\right)^k \left(\frac{q_1}{\tilde{q}}\right)^{N-k} \right) \\ = \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln \left(\alpha p_0^k q_0^{N-k} + (1-\alpha) p_1^k q_1^{N-k} \right) \\ - \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln (\tilde{p})^k (\tilde{q})^{N-k} \end{aligned}$$

Since

$$\sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln (\tilde{p})^k (\tilde{q})^{N-k} =$$

$$= \ln(\tilde{p}) \sum_{k=0}^{N} k C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) + \ln(\tilde{q}) \sum_{k=0}^{N} (N-k) C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k})$$

$$= \ln(\tilde{p}) \sum_{k=0}^{N} k C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) - \ln(\tilde{q}) \sum_{k=0}^{N} k C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k})$$

$$= \ln\left(\frac{\tilde{q}}{\tilde{p}}\right) \sum_{k=1}^{N} k C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) = \ln\left(\frac{\tilde{q}}{\tilde{p}}\right) \sum_{k=1}^{N} N C_{N-1}^{k-1} (p_0^k q_0^{N-k} - p_1^k q_1^{N-k})$$

$$= \ln\left(\frac{\tilde{q}}{\tilde{p}}\right) \left(\sum_{k=1}^{N} N p_0 C_{N-1}^{k-1} p_0^{k-1} q_0^{N-k} - \sum_{k=1}^{N} N p_1 C_{N-1}^{k-1} p_1^{k-1} q_1^{N-k} \right)$$

$$= \ln\left(\frac{\tilde{q}}{\tilde{p}}\right) \sum_{k=1}^{N} N C_{N-1}^{k-1} (p_0^k q_0^{N-k} - p_1^k q_1^{N-k})$$

$$= \ln\left(\frac{\tilde{q}}{\tilde{p}}\right) \left(\sum_{k=1}^{N} N p_0 C_{N-1}^{k-1} p_0^{k-1} q_0^{N-k} - \sum_{k=1}^{N} N p_1 C_{N-1}^{k-1} p_1^{k-1} q_1^{N-k} \right)$$

$$= (N p_0 - N p_1) \ln\left(\frac{\tilde{q}}{\tilde{p}}\right),$$

we have

$$f'(\alpha) = \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln\left(\alpha p_0^k q_0^{N-k} + (1-\alpha) p_1^k q_1^{N-k}\right)$$

$$-N(p_0 - p_1) \ln\left(\frac{\tilde{q}}{\tilde{p}}\right).$$

Hence

$$f'(1) = \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln (p_0)^k (q_0)^{N-k}$$

$$-N(p_0 - p_1) \ln\left(\frac{\tilde{q}}{\tilde{p}}\right)$$

$$= N(p_0 - p_1) \ln\left(\frac{p_0}{q_0}\right) - N(p_0 - p_1) \ln\left(\frac{\tilde{q}}{\tilde{p}}\right) = (N p_0 - N p_1) \ln\left(\frac{p_0 \tilde{q}}{q_0 \tilde{p}}\right).$$

and

$$f'(0) = \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln (p_1)^k (q_1)^{N-k}$$

$$= (N p_0 - N p_1) \ln\left(\frac{p_1 \tilde{q}}{q_1 \tilde{p}}\right)$$

From $p_0 < \tilde{p} < p_1$ follows $\frac{p_0\tilde{q}}{q_0\tilde{p}} < 1$ and $\frac{p_1\tilde{q}}{q_1\tilde{p}} > 1$ (since $\frac{p_0+q_0}{p_0} = \frac{1}{p_0} > \frac{1}{\tilde{p}} = \frac{\tilde{p}+\tilde{q}}{\tilde{p}} \Rightarrow \frac{q_0}{p_0} > \frac{\tilde{q}}{\tilde{p}}$). Thus $f'(1) > 0,\ f'(0) < 0$.

If $p_0 < p_1 \le \tilde{p}$ then from $f'(0) > 0$ and $f''(\alpha) = \tilde{E}(Z(p_0) - Z(p_1))^2 \frac{1}{Z^\alpha} > 0$ follows $f'(\alpha) > 0,\ \alpha \in [0,1]$. Similarly one can prove $f'(\alpha) < 0,\ \forall\alpha$, if $\tilde{p} \le p_0 < p_1$.

Corollary. If $\tilde{p} = \tilde{q} = \frac{1}{2}$ and $p_1 = 1 - p_0$, then

$$f'(\alpha) = \sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - p_1^k q_1^{N-k}) \ln\left(\alpha p_0^k q_0^{N-k} + (1-\alpha) p_0^{N-k} q_0^k\right)$$

$$-N(p_0 - p_1)\ln(1) = \sum_{k=0}^{N} C_N^{N-k} (p_0^k q_0^{N-k} - q_0^k p_0^{N-k}) \ln\left(\alpha p_0^k q_0^{N-k} + (1-\alpha) p_0^{N-k} q_0^k\right)$$

$$= -\sum_{k=0}^{N} C_N^k (p_0^k q_0^{N-k} - q_0^k p_0^{N-k}) \ln\left((1-\alpha) p_0^k q_0^{N-k} + \alpha p_0^{N-k} q_0^k\right) = -f'(1-\alpha),$$

$$f'(\frac{1}{2}) = 0 \ and \ \alpha^* = \frac{1}{2}.$$

Hence we have

3 The more general approach

Let $(P_\theta, \theta \in \Theta)$ be a family of probability measures and $Z(\theta) = \frac{dP_\theta}{d\tilde{P}}$. It has the natural extension to family $(P_\nu, \nu \in \mathcal{P}(\Theta))$ with $Z(\nu) = \frac{dP_\nu}{d\tilde{P}}$. Let $\tilde{\theta}(\omega)$ be a estimation of parameter θ and $\int_\Omega (\tilde{\theta}(\omega) - \theta)^2 P_\theta(d\omega) = E_\theta(\tilde{\theta} - \theta)^2$. $W(\tilde{\theta}) = \max_{\theta\in\Theta} E_\theta(\tilde{\theta} - \theta)^2$ is efficiency criteria of $\tilde{\theta}$. Let $(\tilde{\theta}^\dagger, \nu^\dagger)$ be a saddle point, i.e.

$$W(\tilde{\theta}^\dagger) = \min_{\tilde{\theta}} \max_{\theta\in\Theta} E_\theta(\tilde{\theta} - \theta)^2 = \int_\Theta E_\theta(\tilde{\theta}^\dagger - \theta)^2 d\nu^\dagger(\theta).$$

Each pair $(\tilde{\theta}, \nu)$ defines the measure on Ω in the following way. The distribution of random variable $\tilde{\theta}$ is the image of measure P_ν by the mapping $\tilde{\theta}$, i.e. $\tilde{\theta}_*(P_\nu)$. We have $Z(\tilde{\theta}_* P_\nu) = \frac{dP(\tilde{\theta}_* P_\nu)}{d\tilde{P}}$. For the saddle point $(\tilde{\theta}^\dagger, \nu^\dagger)$ we denote $Z^\dagger = Z(\tilde{\theta}^\dagger_* P_{\nu^\dagger}),\ P^\dagger = P(\tilde{\theta}^\dagger_* P_{\nu^\dagger})$. Now the problem is

$$\max_{\tilde{E}V = X_0'} \tilde{E} Z^\dagger U(V).$$

For the family $P_\theta(\omega) = \sum_{\omega\in\Omega} \theta^{\sharp H(\omega)} (1-\theta)^{\sharp T(\omega)}, \theta \in \Theta \subset [0,1]$ since $\tilde{\theta}_* P_\theta = \sum_x \theta^{\#H(x)} (1-\theta)^{\#T(x)} \delta_{\tilde{\theta}(x)}$ we have that

$$Z(\tilde{\theta}_* P_\theta) = \sum_{x\in\Omega} Z(\tilde{\theta}(x)) \theta^{\sharp H(x)} (1-\theta)^{\sharp T(x)}.$$

More general for $\tilde{\theta}_* P_\nu = \sum_x \delta_{\tilde{\theta}(x)} \nu(\theta^{\#H(x)}(1-\theta)^{\#T(x)})$ the similar equality

$$Z(\tilde{\theta}_* P_\nu) = \sum_{x \in \Omega} Z(\tilde{\theta}(x)) \nu(\theta^{\sharp H(x)}(1-\theta)^{\sharp T(x)})$$

takes place. It is clear that

$$W(\tilde{\theta}) = \max_{\theta} \sum_{x \in \{H,T\}^N} (\tilde{\theta}(x) - \theta)^2 \theta^{\sharp H(x)}(1-\theta)^{\sharp T(x)}.$$

The saddle point for the case $\Theta = [0,1]$ calculated explicitly [1]

$$\theta^\dagger(x) = \frac{\#H(x) + \sqrt{N}/2}{N + \sqrt{N}},\ \nu^\dagger(d\theta) = \frac{\Gamma(\sqrt{N})}{\Gamma^2(\frac{\sqrt{N}}{2})}(\theta(1-\theta))^{\frac{\sqrt{N}}{2}-1} d\theta.$$

Hence

$$Z^\dagger = \frac{\Gamma(\sqrt{N})}{\Gamma^2(\sqrt{N}/2)} \sum_{x \in \Omega} Z\left(\frac{\#H(x) + \sqrt{N}/2}{N + \sqrt{N}}\right) \int_0^1 \theta^{\#H(x) + \frac{\sqrt{N}}{2} - 1}(1-\theta)^{\#T(x) + \frac{\sqrt{N}}{2} - 1} d\theta.$$

The solution of such utility maximization problem for logarithmic utility will be $V^* = X_0' Z^\dagger$. This solution for the cases $N = 1, 2$ coincides with solutions of Example 1 and Example 2.

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A Martingale Characterization of the General Solution of Quadratic Functional Equation

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Abstract. We describe the class of functions $f = (f(x), x \in R)$, for which the process $f(W_t) - Ef(W_t)$ is a martingale. This result is applied to give a martingale characterization of the general solution of the quadratic functional equation.

1.Introduction. It is well known that if for a function $f = (f(x), x \in R)$ the transformed process $(f(W_t), t \geq 0)$ of a Brownian Motion W is a martingale, then f is a linear function (see, e.g. [2]). In this paper we describe the class of function f for which the process $f(W_t) - Ef(W_t)$ is a martingale. We prove that the process

$$f(W_t) - Ef(W_t), \quad t \geq 0$$

is a right-continuous martingale if and only if the function f is a square trinomial , i.e., $f(x) = ax^2 + bx + c$ for some constants a, b and c. Besides, we show that if $f(W_t) - Ef(W_t)$ is a martingale (without assuming the regularity of paths), then $f(x)$ is equal to some square trinomial almost everywhere with respect to the Lebesgue measure.

We use this result to characterize solutions of quadratic functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y) \quad \text{for all} \quad x,y\in R, \tag{1}$$

the general measurable solution of which (see, e.g., [5]) is the function $f(x)=ax^2, a\in R$. We give an equivalent characterization of equation (1) in terms of martingales, which gives also a probabilistic proof of this assertion.

2. The Main Result. Let $W=(W_t, t\geq 0)$ be a standard Brownian Motion defined on a probability space $(\Omega,\mathcal{F},\mathcal{P})$ with filtration $F=(\mathcal{F}_t, t\geq 0)$ satisfying the usual conditions of right-continuity and completeness.

Theorem 1. Let $f=(f(x), x\in R)$ be a function of one variable, such that $f(W_t)$ is integrable for every $t\geq 0$.

a) If the process

$$M_t=f(W_t)-Ef(W_t), \quad t\geq 0,$$

is a right-continuous (P- a.s.) martingale, then the function f is of the form

$$f(x)=ax^2+bx+c \quad \text{for some constants} \quad \alpha, b \quad \text{and} \quad c\in R. \tag{2}$$

b) If the process M_t is a martingale, then $f(x)$ coincides with the function ax^2+bx+c (for some constants $a,b,c\in R$) almost everywhere with respect to the Lebesgue measure.

Proof. a) Let

$$g(t)\equiv Ef(W_t)=\int_R f(y)\frac{1}{\sqrt{2\pi t}}e^{-\frac{y^2}{2t}}dy.$$

Since $E|f(W_t)|<\infty$ for all $t\geq 0$ the function $g(t)$ will be continuous for any $t>0$. Since M_t is right-continuous (P-a.s.) and $g(t)$ is continuous, the process $f(W_t)$ will be also right-continuous P-a.s.. If outside a null set the process $f(W_t)$ is right-continuous then each point $x\in R$ is reached by the process W_t from above at some finite time and from below at some other finite time. This implies that the function $f(x)$ is continuous (see [2]).

Let

$$F(t,x)=f(x)-g(t), \quad t\geq 0, x\in R.$$

Since $F(t,W_t)$ is a martingale, we have that

$$F(t,W_t)=f(W_t)-g(t)=E(f(W_T)/\mathcal{F}_t)-g(T) \tag{3}$$

$P-a.s.$ for all $t \leq T$. Let

$$u(t,x) = E(f(W_T)/W_t = x).$$

It is well known that $u(t,x)$ will be of the class $C^{1.2}$ on $(0,T)\times R$ and satisfies the backward Kolmogorov equation (see, e.g. [3])

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < t < T, x \in R. \tag{4}$$

By the Markov property of W

$$u(t,W_t) = E(f(W_T)/\mathcal{F}_t)$$

and from (3) we have that P-a.s.

$$f(W_t) = u(t,W_t) - g(T) + g(t).$$

Therefore,

$$\int_R \left|f\left(x\right) - u\left(t,x\right) + g(T) - g(t)\right| \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}} dx = 0$$

which implies that for any $0 < t \leq T$

$$f\left(x\right) = u\left(t,x\right) - g(T) + g(t) \quad a.e. \tag{5}$$

with respect to the Lebesgue measure. Since f and u are continuous, we obtain that for any $0 < t \leq T$

$$u\left(t,x\right) = f\left(x\right) + g(T) - g(t) \quad \text{for all} \quad x \in R. \tag{6}$$

This implies that $g(t)$ is differentiable, $f(x)$ is two-times differentiable and it follows from (4) and (6) that

$$\frac{1}{2} f''(x) = g'(t). \tag{7}$$

Since the left-hand side of (7) does not depend on t and the right-hand side on x, both parts of (7) are equal to a constant. Therefore, we obtain that

$$f''(x) = 2a \quad \text{and} \quad g'(t) = a \quad \text{for some} \quad a \in R. \tag{8}$$

The solutions of these equations are

$$f(x) = ax^2 + bx + c \quad \text{and} \quad g(t) = at + c \tag{9}$$

respectively, for some $a, b, c \in R$.

b) Let

$$\tilde{f}(x) = u(t_0, x) + g(t_0) - g(T).$$

for some $t_0 > 0$. It follows from (5) that

$$\lambda(x : f(x) \neq \tilde{f}(x)) = 0, \tag{10}$$

where λ is the Lebesgue measure and by definition of $u(t, x)$ the function $\tilde{f}(x)$ is continuous (moreover, it is two-times differentiable). It follows from (10) that $P(f(W_t) = \tilde{f}(W_t)) = 1$ for any $t \geq 0$ and since $Ef(W_t) = E\tilde{f}(W_t)$, we obtain that the processes $M_t = f(W_t) - Ef(W_t)$ and $\tilde{M}_t = \tilde{f}(W_t) - E\tilde{f}(W_t)$ are equivalent, which implies that the process $\tilde{M}_t$ is a continuous martingale. Therefore, it follows from part a) of this theorem that $\tilde{f}(x)$ is of the form (2) and hence, $f(x)$ coincides with the function (2) almost everywhere with respect to the Lebesgue measure. □

Remark. The converse is also true. If the function f is of the form (2), then

$$f(W_t) = aW_t^2 + bW_t + c, \quad Ef(W_t) = at + c$$

and the process $f(W_t) - Ef(W_t) = a(W_t^2 - t)$ is a martingale.

Corollary 1. Let $f = (f(x), x \in R)$ be a function of one variable.

a) If the process $(f(W_t), \mathcal{F}_t, t \geq 0)$ is a right-continuous martingale, then

$$f(x) = bx + c \quad \text{for all} \quad x \in R \tag{11}$$

for some constants b, c.

b) If the process $(f(W_t), \mathcal{F}_t, t \geq 0)$ is a martingale, then $f(x) = bx + c$ almost everywhere with respect to the Lebesgue measure for some constants $b, c \in R$.

Proof. If the process $f(W_t)$ is a martingale, then $g(t) = Ef(W_t)$ is constant and the coefficient a in (9) is equal to zero. Therefore, this corollary follows from Theorem 1. □

3. Application to Quadratic Functional Equations. It was proved in [4] that if the function $f = (f(x), x \in R)$ is a measurable solution of the Cauchy additive functional equation

$$f(x+y) = f(x) + f(y), \quad \text{for all} \quad x, y \in R,$$

then the transformed process $(f(W_t), t \geq 0)$ is a right-continuous martingale, which (by Corollary 1 of Theorem 1) implies that f is a linear function. Here we propose a similar characterization of solutions of the quadratic functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \quad \text{for all} \quad x, y \in R. \tag{12}$$

It is well known (see, e.g., [5]), that the general measurable solution of equation (12) is the function $f(x) = ax^2, a \in R$. Using Theorem 1 we give an equivalent characterization of equation (12) in terms of martingales , which gives also a probabilistic proof of this assertion.

Theorem 2. Let the function $f = (f(x), x \in R)$ be non-negative (or non-positive) for all $x \in R$. The following assertions are equivalent:

i) the function $f = (f(x), x \in R)$ is a measurable solution of (12),

ii) $f = (f(x), x \in R)$ is an even function with $f(0) = 0$ and such that the process

$$N_t = f(W_t) - Ef(W_t), \quad t \geq 0,$$

is a right-continuous martingale,

iii) the function f is of the form

$$f(x) = ax^2, \tag{13}$$

for some constant $a \in R$.

Proof. $i) \rightarrow ii)$ It is evident that if f is a solution of (12) then $f(0) = 0$ and $f(x) = f(-x)$ for all $x \in R$. Therefore, since $f(x)$ is non-negative or non-positive, expectations below have a sense and for any random variable ξ with symmetric distribution

$$Ef(\xi - x) = Ef(x - \xi) = Ef(\xi + x). \tag{14}$$

Substituting $x = W_t - W_s$ and $y = x + W_s$ in equation (12) we have that

$$f(W_t) + f(W_t - 2W_s - x) = 2f(W_t - W_s) + 2f(x + W_s). \tag{15}$$

Since $E(W_t - 2W_s)^2 = t$ for $s \leq t$, the random variables W_t and $W_t - 2W_s$ have the same normal distributions and hence

$$Ef(x + W_t) = Ef(x + 2W_s - W_t) = Ef(W_t - 2W_s - x). \tag{16}$$

Therefore, taking expectations in (15) we obtain

$$Ef(x + W_t) = Ef(W_t - W_s) + Ef(x + W_s) \quad s \leq t. \tag{17}$$

Let $g(t) = Ef(W_t)$. Then $Ef(W_t - W_s) = g(t - s)$ and it follows from (17) (for $x = 0$) that g satisfies the Cauchy additive functional equation

$$g(t) = g(t - s) + g(s), \quad s \leq t$$

on R^+. As it is well known (see, e.g., [1]) that any bounded from bellow (or from above) solution of this equation is of the form $g(t) = ct$ for some constant $c \in R$. Therefore, $f(W_t)$ is integrable for any $t \geq 0$ and if we take $s = 0$ in (17) we obtain

$$f(x) = Ef(x + W_t) - Ef(W_t) = \int_R f(x + y)\frac{1}{\sqrt{2\pi t}}e^{-\frac{y^2}{2t}}dy - Ef(W_t) =$$

$$= \int_R f(y)\frac{1}{\sqrt{2\pi t}}e^{-\frac{(y-x)^2}{2t}}dy - Ef(W_t), \tag{18}$$

which implies that f is continuous.

Taking now conditional expectations in (15) for $x = 0$, using the independent increment property of W and then equality (17) we have that P-a.s.

$$E(f(W_t)|F_s) + E(f(W_t - 2W_s)|F_s) = 2E(f(W_t - W_s)|F_s) + 2f(W_s) =$$

$$= 2Ef(W_t - W_s) + 2f(W_s) = 2Ef(W_t) + 2f(W_s) - 2Ef(W_s). \tag{19}$$

On the other hand, it follows from (14) that P-a.s.

$$E(f(W_t - 2W_s)|F_s) = E(f(W_t - W_s - W_s)|F_s) = \big(Ef(W_t - W_s - x)\big)_{W_s} =$$

$$= \big(Ef(W_t - W_s + x)\big)_{W_s} = E(f(W_t - W_s + W_s)|F_s) = E(f(W_t)|F_s). \tag{20}$$

Therefore the martingale equality

$$E(f(W_t) - Ef(W_t)|F_s) = f(W_s) - EfW_s), \quad P - \text{a.s.},$$

follows from equations (19) and (20). Thus, the process $N = (f(W_t) - Ef(W_t), t \geq 0)$ is a martingale with P-a.s. continuous paths.

ii) $\rightarrow$ *iii)*. Since $f(W_t) - Ef(W_t)$ is a martingale, Theorem 1 implies that the function f should be of the form

$$f(x) = ax^2 + bx + c, \quad a, b, c \in R.$$

Since f is even we have that $b = 0$ and $c = 0$ since $f(0) = 0$. Thus, $f(x) = ax^2$ for some $a \in R$.

The proof of implication *iii)* $\rightarrow$ *i)* is evident.

□

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ციფრული გარდაქმნის სტრატეგიები მცირე და საშუალო ბიზნესში საქართველოს მაგალითზე

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შესავალი

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5 მცირე ან საშუალო ბიზნესის დამფუძნებელი/წილის მფლობელი, ხოლო 15 სხვადასხვა მენეჯერულ პოზიციაზე მომუშავე თანამშრომელი.

კვლევის დიზაინი

თვისობრივი კვლევის ჩასატარებლად გამოყენებულ იქნება სიღმისეული ინტერვიუს მეთოდი. სიღმისეული ინტერვიუს მიზანია, გავიგოთ რესპოდენტების მაქსიმალურად დეტალური მოსაზრებები საკვლევ საკითხთან დაკავშირებით. ინტერვიუს დროს გამოყენებულ იქნება სიღრმისეული ინტერვიუს სამივე ფაზა, პრეინტერვიუ, კერძოდ ინტერვიუ და პოსტინტერვიუ. ინტერვიუ სადისკუსიო გეგმის მიხედვით წარიმართება, მაგრამ შეიძლება დიალოგის დროს წამოიჭრას ისეთი საკითხები რომელიც არ არის გეგმაში შეტანილი თუმცა აუცილებლად დაეხმარება კვლევას. აუცილებლად გათვალისწინებულ უნდა იქნას რომ დიდად არ მოხდეს საკვლევი თემიდან გადახვევა და ინტერვიუერის მიერ უნდა მოხდეს ამ პროცესის სრული გაკონტროლება.

სიღრმისეული ინტერვიუს მეშვეობით უნდა მოვახდინოთ მაქსიმალურად ამომწურავი ინფორმაციის შეგროვება. მსგავსი ტიპის კვლევები მოგვცემს საშუალებას, რომ უკეთ შევისწავლოთ ქართულ ბაზარზე არსებული მცირე და საშუალო ბიზნესი, მათი მუშაობის ორგანიზაციული სისტემა, უშუალოდ თანამშრომლების თვალით დანახული გამოწვევები. მიზანშეწონილი იქნება საკვლევი საკითხის მომავალში რაოდენობრივი კვლევით გამყარება.

კვლევის შედეგების ანალზი

თვისობრივი კვლევის ფარგლებში სიღრმისეული ინტერვიუ ჩაუტარდა ოც ადამიანს, მცირე ან საშუალო ბიზნესის 5 დამფუძნებელს/წილის მფლობელს და 15 მენეჯერულ პოზიციაზე მომუშავე თანამშრომელს. კვლევის შედეგები გაანალიზებული იქნა კვლევის ამოცანებთან მიმართებაში.

პირველი ამოცანის მიხედვით გამოკითხულთა უმრავლესობა აცნობიერებს ციფრული ტრანსფორმაციის მნიშვნელობას. გამოკითხვისას ნათელი გახდა, რომ პანდემიით გამოწვეულმა კრიზისმა და სოციალურმა მდგომარეობამ ბიზნესს უბიძგა ციფრული განვითარებისთვის. ინტერვიუს დროს გამოკითხულთა თითქმის 100 % აღიარებს რომ საქართველოში მცირე და საშუალო ბიზნესს ყველაზე მეტად თანამედროვე ცონდა და ტექნოლოგიები აკლიათ - გაციფრულების უმნიშვნელოვანესი უპირატესობაა ბიზნესის ანალიზი და გაზომვადობაა, რომლის გარეშეც წარმატებაზე და განვითარებაზე ფიქრიც კი აბსურდია. სიღმისეული ინტერვიუს დროს გამოიკვეთა გარემოებები რომლებიც თვითონ რესპოდენტებმა წამოჭრეს, მაგალითად სავაჭრო სექტორში გამოკითხულთა უმრავლესობა თვლის რომ ონლაინ მაღაზიების შექმნა აუცილებელია. გამოკითხულთა შორის მხოლოდ 2 მა ადამიანის აზრით ბიზნესის ციფრული გადრაქმნის აუცილებლობა გამოიწვია

პანდემიამ და მასთან გამოწვეულმა შეზღუდვებმა და ამ შეზღუდვების შემსუბუქებასთან ერთად ბიზნეს პროცესები დაუბრუნდება ძველ მდგომარეობას.

გამოკითხულთა ის დიდი ნაწილი რომელიც აცნობიერებს ციფრული ტრანსფორმაციის მნიშვნელობას და დადებითად აფასებს მას, აღიარებს რომ ტექნოლოგიური ინოვაციები ბიზნეს სექტორში დაახლოებით 30 წელიწადია ეტაპობრივად ვითარდება და დღევანდელ დღემდე ისე დაიხვეწა რომ მათი უგულვებელყოფა შეუძლებელია. მათი აზრით ციფრული ტექნოლოგიის უმთავრესი უპირატესობობა ბიზნესის პროცესების მართვის გამარტივებაა, დათვლადობა, ბიზნეს პროცესების ავტომატიზირება და ხარჯების შემცირებაა. გამოკითხულთა ნახევარზე მეტი აცნობიერებს რომ ციფრულ პროდუქტებს უკვე მათი მომხმარებლებიც ითხოვენ და თანამედროვე სტრატეგიების გარეშე უბრალოდ ვეღარ იქნებიან კონკურენტუნარიანები.

კვლევის მესამე ამოცანა ეხება მცირე და საშუალო ბიზნესის ციფრულ გარდაქმნასთან დაკავშირებულ პრობლემებს. სიღრმისეულ ინტერვიუში მონაწილე რესპოდენტების 100 % თანხმდება იმაზე რომ სწორედ ეს მესამე ამოცანაა ყველაზე მნიშვნელოვანი და სენსიტიური საქართველოს ბაზარზე არსებული მცირე და საშუალო ზომის კომპანიებისთვის. ციფრული ტრანსფორმაციის დროს მრავალი გამოწვევის წინაშე დგანან მცირე ზომის ბიზნესები. პირველ რიგში, ისინი შეზღუდულები არიან რესურსებში რის გამოც უჭირთ თანამედროვე ტექნოლოგიების დანერგვა მათ ბიზნეს პროცესებში. რესპოდენტები აღნიშნავენ რომ მათვის ერთ-ერთი გამოწვევაა კვალიფიცირებული მუშახელის მოძიება ან თანამშრომელთა გადამზადება რომლებიც შეძლებენ ახალ ტექნოლოგიებთან მუშაობას.

უნდა აღინიშნოს, რომ კვლევის მომზადებისას გამოვლინდა შემდეგი საინტერესო გარემოება: ციფრულ გარდაქმნასთან დაკავშირებული კვლევები ჩატარებულია მხოლოდ დიდ კომპანიებში, აღსანიშნავია რომ მსგავსი ტიპის კომპანიებისათვის რესურსების დეფიციტი არ წარმოადგენს პრიორიტეტულ პრობლემას და ასეთი კომპანიებისათვსი სხვა გამოწვევები დგას წინა პლანზე (მაგ: გარდაქმნისათვის და ცვლილებებისთვის საჭირო სტრუქტურულ/ სტრატეგიული მოქნილობა სხვადასხვა პარამეტრების გათვალისწინებით), რასაც ვერ ვიტყვით მცირე ზომის კომპანიების შემთხვევაში მათი პრიორიტეტი სწორედ ცვლილებებისადმი მოქნილობა და ახალ სტრუქტურასთან სწრაფი ადაპტირებაა, თუმცა მათთვის საკმაოდ რთულ გამოწვევას წარმოადგენს რესურსების მოზიდვა/ მობილიზება. ყოველივე ზემოაღნიშნულის გათვალისწინებით, საინტერესო იქნება რაოდენობრივი კვლევის ჩატარება ციფრულ ტრანსფორმაციაზე მცირე და საშუალო ბიზნეს სექტორში.

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Incorporating Machine Learning Models in Valuation of Hedging Strategies

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Introduction

This paper follows work introduced in [15] based on gold price risk management problem and valuation of investment in gold widgets production industry. Previous work describes valuation of hedging strategies that can be used in managing risks inherent in investment projects using protective put to hedge against gold price risk. Because marketed puts on all investments and specific strike prices might not be found, a replicating portfolio principle is used to mimic the payoff from the put with any strike chosen.

This paper further improves value from replicating protective put strategy by varying the strike price of the put option according to gold price weekly predictions by machine learning methods. This, in turn, reduces cost of rebalancing the portfolio, i.e., is the cost of the put option itself and, finally, the total cost of the portfolio that is used as the strike price of the real option in the investment project.

Value of protective put with fixed at-the-money strike price

This section follows practical example described in [12] with Georgian producer of gold widgets, Zarapkhana and its gold price risk hedging problem. Opposed to Robust Valuation techniques described in [1] and [12], where project valuation is required under incomplete markets scenario and it is focused on risk mitigation of non-tradable assets such as widgets produced by local manufacturers, this paper focuses on risk management of Gold price risk as an input cost for producing widgets. Based on statistical analysis and Monte Carlo Simulation discussed in [9] and sample data in [15] next year's expected values and standard deviations of gold and widget prices are given in the table below:

widget	**E[η]**	**$87.66**	*gold*	**E[S]**	**$42.03**
	σ	**54%**		**σ**	**29%**

For simplicity, let discount rate, r, equal to 2% per year. Under one-year period investment horizon present value (PV) of cash flow from widget production is given as:

$$PV = \frac{\eta^1 - S^1}{r} = \frac{87.66 - 42.03}{1 + 0.02} = 44.74$$

Next investment value of the project is extended to cover the opportunity to hedge the gold price with put option using Real Options Analysis discussed in [2], [3], and [4]. Following [15], i.e., example of Zarapkhana, value of investing in the project using protective put strategy with at the money strike price of USD 40, resulted into value-add of 42.42 per gram. Thus, PV of the project with real option became:

$$\boldsymbol{PV = DCF + Real\ Option\ Value = 44.74 + 42.42 = 87.16}$$

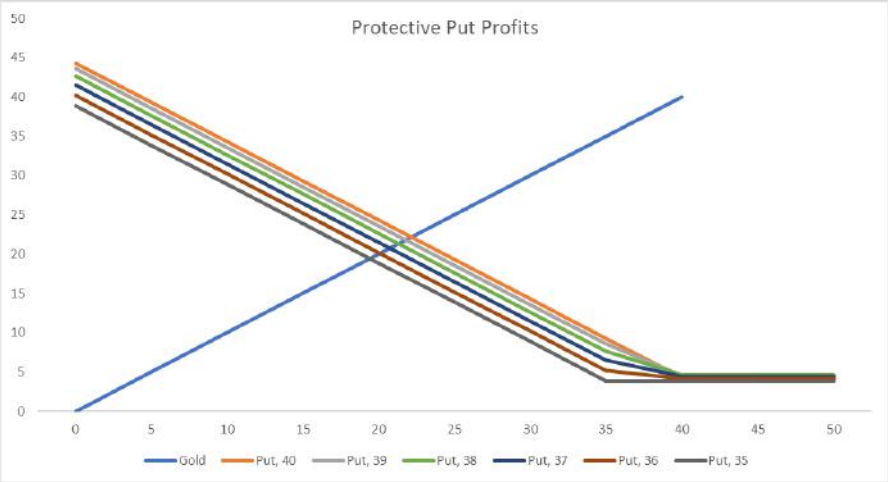
Addition value from using protective put hedge on gold prices was valued at USD 42.42 per gram, which is almost as much as its original PV obtained by standard DCF analysis.

Using Machine Learning Models to predict Gold price

Previous section calculated value of investment project with real option of using protective put strategy with a fixed strike price equal to current market price of gold. In contrast, strike price of put option can be further optimized by having a particular view about future price of gold. Moreover, protective put strategy employed in previous section and as described in [15] uses replicating portfolio principle to mimic the returns of a put option for a given / any strike price. For example, using sample data from the same example, total cost and put price calculated as the cost of weekly rebalancing the portfolio is illustrated in the table below:

Total Cost	*48.72*	*48.49*	*48.10*	*47.53*	*46.87*	*46.20*
Put Price	*4.30*	*4.56*	*4.62*	*4.47*	*4.19*	*3.86*
Strike	*40*	*39*	*38*	*37*	*36*	*35*

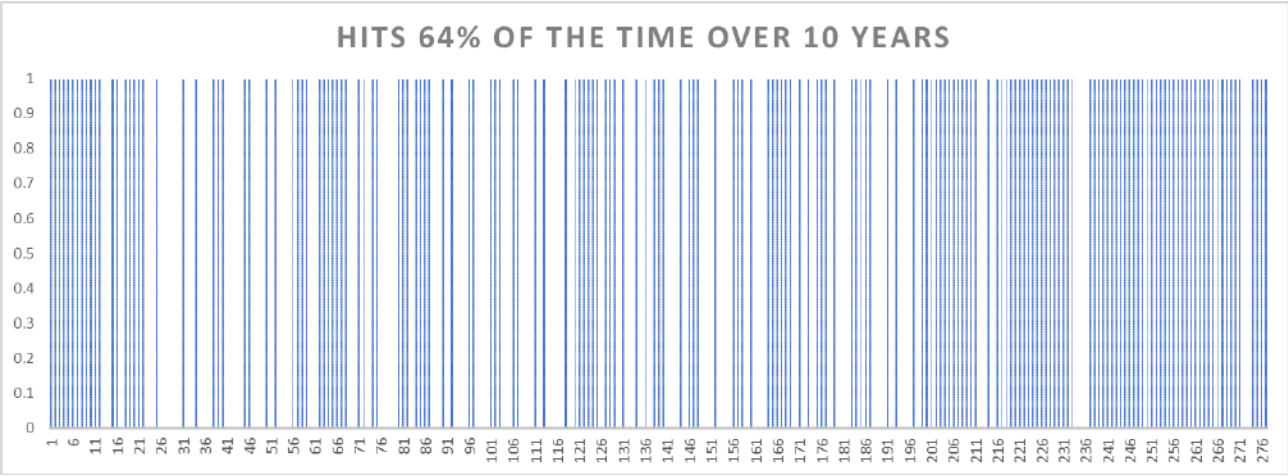
In addition, profit diagrams from protective put strategy with fixing different strike prices are shown below:



Ultimately, optimal strike price will depend on the future gold price. In line with weekly rebalancing of protective put portfolio, one can come up with weekly updated predictions of gold price and use that as a strike price of the put. In this section, basic Machine Learning (ML) Models from [16] such as:

- K-nearest neighbor (KNN)
- Gradient Boosting Trees (GBT)
- Support Vector Machines (SVM)
- Gaussian Processes Regression (GRP)

Again, using sample data in [15], ensemble of above ML models is used to predict weekly prices of gold. An example of such predictions compared with actual gold prices over period of 10 years is illustrated below:

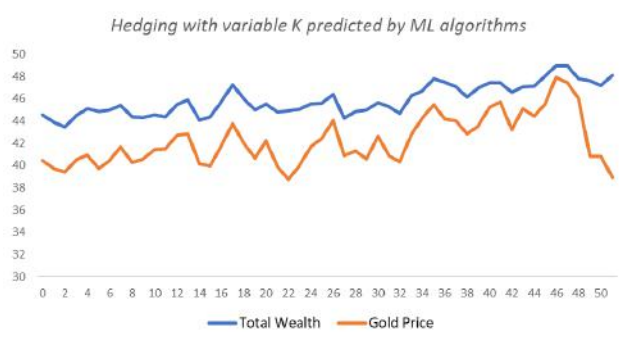


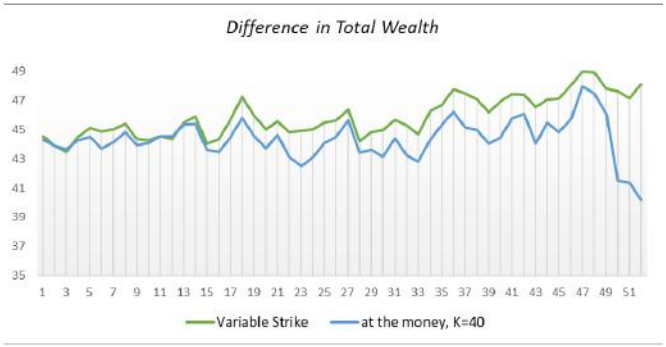
Above graph shows that 64% of the time predictions were accurate about the direction of gold price increase or decrease. In the next section, weekly predictions of gold price will be used as the strike price for put option in hedging strategy.

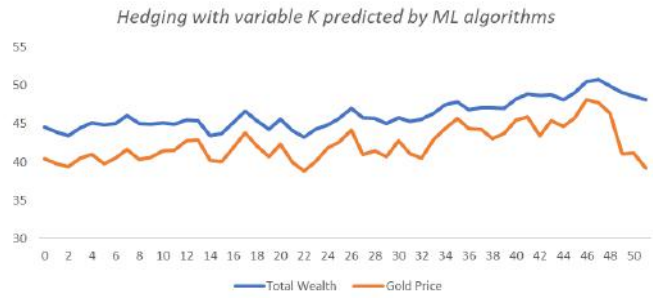
Value of protective put with variable strike price

Now, it's important to compare how well predicted gold price work as a variable strike price for protective put strategy against fixed strike price. From example in [15], total wealth from protective put strategy and gold price simulation discussed in [9] is shown below:

Using the same simulation example but varying the strike price given by ensemble of ML model predictions, total wealth and gold price graphs become:







Further proceeding with real option valuation of this investment project, additional increase in optional values is gained in amount of 13.42 per gram. Final investment project value is given by:

$$\boldsymbol{PV = DCF + Real\ Option\ Value = 44.74 + 55.84 = 113.88}$$

Conclusion

Based on previous work in [15], investment project valuation has been continued to incorporate weekly predictions of gold price using an ensemble of Machine Learning Algorithms like KNN, GBT, SVM, GPR as the strike price for protective put hedging strategy. This in turn provided greater final value of investment project at hand.

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